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DEVELOPMENT OF ARITHMETIC AS A SCHOOL SUBJECT

By WALTER SCOTT MONROE

PROFESSOR OF SCHOOL ADMINISTRATION, KANSAS STATE NORMAL SCHOOL

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LETTER OF TRANSMITTAL.

DEPARTMENT OF THE INTERIOR,
BUREAU OF EDUCATION,
Washington, February 9, 1917.

SIR: The courses of study in our elementary and secondary schools have been accepted largely on tradition and without intelligent criticism as to their adaptation to modern conditions and needs or their educational value. In some subjects much new material has been introduced from time to time, while conservative tendencies have prevented the elimination of the old and outgrown, thus resulting in a congestion of material detrimental to the interests of the subjects and to the proper balance of the entire curriculum. This has been especially true of arithmetic as a school subject. There is now, however, a tendency everywhere to reorganize the curriculum both of elementary and of secondary schools, to the end that each subject may have its proper share of time and attention. That this work of readjustment in arithmetic may be done intelligently, it is very desirable that those engaged in it may have as complete knowledge as possible of the development of this subject and of past as well as present practice in regard to it. I therefore recommend for publication as a bulletin of the Bureau of Education the manuscript transmitted herewith, giving a history of arithmetic in the schools of the United States.

Respectfully submitted.

P. P. CLAXTON,
Commissioner.

The SECRETARY OF THE INTERIOR.

PREFACE.

The arithmetic with which the American schoolboy of the twentieth century wrestles differs in many respects from the "cyphering" which was truly a stumbling block to many a child in colonial days. Not only have there been significant changes in the subject matter of arithmetic, but also in the aim of instruction, in the place of arithmetic in the plan of education, and in the methods of teaching the subject. In fact many of the distinguishing characteristics of arithmetic as a twentieth century school subject are products of the nineteenth century. It has been the purpose of this investigation to trace in some detail the development of arithmetic as a school subject and the methods of teaching it in the United States, and to show the influence of Warren Colburn in stimulating and directing this development.

It is a pleasure to acknowledge indebtedness to those who inspired and encouraged this research and to those who have assisted in making accessible the sources; in particular, Prof. W. W. Charters, of the University of Missouri, who first mentioned the problem, and Profs. S. C. Parker and G. W. Myers, of the University of Chicago, for their direction and helpful criticism.

WALTER SCOTT MONROE.

EMPORIA, KANS.

DEVELOPMENT OF ARITHMETIC AS A SCHOOL SUBJECT.

PART I. THE CIPHERING BOOK PERIOD: FROM THE BEGINNING IN THE COLONIES UP TO 1821.

Chapter I.

THE PLACE OF ARITHMETIC IN COLONIAL EDUCATION.

At the time of the colonization of America in the first half of the seventeenth century, arithmetic was not considered essential to a boy's education unless he was to enter commercial life or certain trades. The instruction in arithmetic was often given in a separate school, called a writing school, or a reckoning school. When arithmetic was taught in the grammar schools it was very rudimentary.[1] Not only was this true, but among the nobility and the aristocracy of the educated, arithmetic was looked upon as "common," "vile," "mechanic," because it was the accomplishment of clerks, artisans, tradesmen, and "others who bore no signs of heraldry." Consequently, it was a subject beneath the dignity of a boy unless he was "less capable of learning and fittest to be put to trades."

Such was arithmetic and the place it occupied in education in Europe at the time when the American colonies were settled. The colonists had grown to maturity in European environment and had been educated in European schools. When they came to America, they brought with them traditions and ideals which influenced their schools and their plan of education.

Dutch New York.—The first settlements in New York were made by the Dutch West India Co., which was chartered in 1621 by the States-General of the United Netherlands. To this company was given a monopoly of Dutch trade within certain areas. The Dutch nation had produced some of the most important commercial centers of Europe, and a nation which had attained such commercial prominence could not neglect arithmetic. When they arrived in America in the interest of a huge commercial enterprise, the Dutch colonists brought with them this attitude toward arithmetic. Prof. Kilpatrick says:

> What might be called the official Dutch program for the colonists was that promulgated by the classis in 1636 in the instruction "for schoolmasters going to the East or West Indies."

[1] F. Watson: The Beginnings of the Teaching of Modern Subjects in England, pp. 286, 308–310.

He is to instruct the youth in reading, writing, cyphering, and arithmetic, with all zeal and diligence, he is also to implant the fundamental principles of the true Christian religion and salvation, by means of catechizing; he is to teach them the customary form of prayers, and also accustom them to pray; he is to give heed to their manners and bring them as far as possible to modesty and propriety.[1]

This official curriculum was not uniformly carried out, according to Prof. Kilpatrick, who has examined the available records with care.[2] In New Amsterdam (now New York) arithmetic was always included in the curriculum, but in the outlying villages, except Albany, which was a commercial center, arithmetic does not appear to have been given a place in the education of the children. In these villages reading, writing, and the catechism made up the curriculum, and arithmetic was not considered a necessary part, perhaps not even a desirable part, of education. This condition emphasizes that arithmetic was considered by the early Dutch colonists to be a practical subject necessary for those engaged in trade and commerce, but not a subject possessing general educational value.

New England.—The New England colonies were settled by the Puritans, who came to America in order that they might secure religious freedom. So strong was their desire to worship according to their beliefs and to perpetuate their church doctrines that they braved the long ocean voyage and the hardships of an unknown and wild land. The sentiment of the first settlements was probably expressed by a member of the Massachusetts company when he said, in 1629: "The propagation of the Gospel is the thing we do profess above all to be our aim in settling this plantation."[3]

A letter written in 1629 describing the colonists of Salem says:

They live unblameable and without reproofe, and demeane themselves in style and curteous towards ye Indians, thereby to draw them to affect our persons and consequentlye our Religion, as alsoe to endeavour to gett some of their children up to reading and consequentlye to religion while they are yonge.[4]

The orders of the General Court of Massachusetts in 1642 and in 1647 emphasize the perpetuation of their religion as the dominant aim in education, and although both reading and writing are mentioned in the order of 1647, no mention is made of arithmetic. A similar law of Connecticut[5] in 1650 likewise makes no mention of arithmetic.

However, school practice can not be deduced with any certainty from official acts. The town records for many of the early settlements have been made accessible in the form of town histories, and these furnish much evidence of what was taught in the first schools of New England. In the Memorial History of Boston there are two

[1] The Dutch Schools of New Netherlands and Colonial New York, p. 220.
[2] Op. cit., pp. 220-221.
[3] Justin Winsor (Ed.): Memorial History of Boston, Vol. IV., p. 235.
[4] Joseph Felt: Annals of Salem, p. 73.
[5] Elsie W. Clews: Educational Legislation and Administration of the Colonial Governments, p. 74.

references to the school practice. The following is quoted from the governor's journal for the year 1645:

Divers free schools were erected, as at Roxbury (for the maintenance whereof every inhabitant bound some house or land for a yearly allowance forever) and at Boston, where they made an order to allow forever 50 pounds to the master and a house, and 30 pounds to an usher, who should also teach to read and write and cipher, and Indians' children to be taught freely, and the charge to be by yearly contribution either by voluntary allowance, or by rate of such as refused, etc.; and this order was confirmed by the general court (blank). Other towns did the like, providing maintenance by several means.[1]

This would lead one to conclude that arithmetic was taught in the "free school" of Boston. Philemon Pormont was the first Boston schoolmaster, beginning his labors in 1635. Littlefield expresses the opinion that, since he was spoken of as "Brother" and not given the title of "Mr.," "he was little more than a writing master" and taught only the elementary branches.[2]

Mr. Daniel Maud was the second teacher, and from his title he probably was a master of arts. According to Littlefield it is probable that both Maud and Pormont taught at the same time, the former giving instruction in the classical studies and the latter in the common branches.[3] If this is true, arithmetic was probably one of the common branches and was taught in the first year of this school's existence. The next reference to the teachers of this school bears the date of 1650.

"It is also agreed on that Mr. Woodmansey, the schoolmaster, shall have fiftye pound p. ann. for his Teaching ye Schollers, and his proportion be made up by ratte." In 1666 the town "agreed with Mr. Dannell Henchman for £4 per ann. to assist Mr. Woodmansey in the Grammar Schoole, and teach Children to wright, the year to begine the 4th of March 65/6."[4]

Arithmetic is not mentioned in this statement, although it may have been included in the writing. The view has been expressed that religion crowded the elementary subjects, particularly arithmetic, out of the school.[5] If this is true, and the purpose of the settlement of Boston together with the general character of the Puritans tends to confirm it, little or no arithmetic was taught in this "free school" of Boston after the first few years of its existence. But in addition to this "free school," which was known as the Boston Latin School, there were other facilities for arithmetical instruction. "In 1667 Will Howard and in 1668 Robert Cannon were licensed to keep a writing school (in Boston) to teach children to write and to keep

[1] Memorial History of Boston, Vol. I, p. 133.
[2] Early New England Schools, p. 64.
[3] Op. cit., p. 67.
[4] Memorial History of Boston, IV, 237.
[5] Ibid., 240-241.

accounts."[1] Also the second public school established in Boston was a writing school, in 1684, in which arithmetic and writing were taught. Thus, even though arithmetic was crowded out of the "free school" in Boston, facilities for giving instruction in arithmetic were provided in these special schools.

Concerning the schools of Salem we find this reference, bearing date of July 20, 1629: "M. Skelton was chosen pastor and Mr. Higginson teacher and they were consecrated to their respective offices."[2] In this statement there is no mention made of what was taught, and the earliest reference to arithmetic being taught in the schools of Salem is the following, which bears the date of 1712:

> As Mr. Emerson had died, a committee are chosen to procure a suitable Grammar schoolmaster to ye instructing of youth in Grammar learning and to fitt them for ye Collidge and also to learn them to write and cypher and to perfect them in reading.[3]

On September 1, 1712, "Nathaniel Higginson commenced the school for reading, writing, and cyphering."[3] Arithmetic as a school subject is mentioned in 1714, 1716, and later. In 1752 this item appears: "After the first of May, all boys who go to the Grammar school must study Latin as well as read, write, and cypher."[4]

From these statements it appears quite certain that arithmetic occupied a fixed place as a school subject in the Salem grammar school in 1712 and after. For the period before 1700 the absence of data makes only speculation possible, but Salem, like Boston, early became a center for trade and commerce. Hence it is probable that facilities existed for giving instruction in arithmetic before 1712.

Dedham, Mass., was founded in 1636, though its history really dates from 1644, which is the date of the establishment of the first school. The following statement shows what was taught in this school in 1653:

> 18 of ye 1 mo. Assemb Job. Kingsbery; Fra. Chickering, Lieft. Fisher, Job. Dwight, Sergt Fisher & Elia Lushes, Pet. Woodward Agreed with Jacob Farrow to keep the Schools to begin 28 of 1 mo. 1653 to 20 L pr an. to be payed in town paye being merchantible at the end of each halfe yeare the one halfe of the said sumes. he undertakes to teach to read English and the Accidence & to write & the knowledg & art of Arithmetick & the rules & practice thereof: this to be p'posed the towne.[5]

A later contract for the year 1656 reads as follows:

> 9 of 11 mo. 1656, Agreed Michaell Metcalfe for to keep the school for the year insuinge, the said Mjchael doe undertake to teach the children that shall be sent to him to reado English and to write.[7]

[1] G. E. Littlefield: Op. cit., p. 81.
[2] Joseph B. Felt: Annals of Salem, p. 100.
[3] Ibid., p. 440.
[4] Ibid., p. 448.
[5] Carlos Slafter: The Schools and Teachers of Dedham, Massachusetts, p. 13.
[6] Carlos Slafter: Op. cit., p. 15.

Only reading and writing are mentioned in this second contract, but in view of what appears to have been the prevailing practice in Dedham and because of additional evidence, it is probable that arithmetic was taught by Michaell Metcalfe. In commenting upon this point, Mr. Slafter says:

> It is hardly to be supposed that Mr. Metcalfe taught only reading and writing, but rather he agreed to teach these at least to all the pupils. There is now in existence the identical arithmetic which he used as a teacher of the school. This book, an enlarged edition of Robert Record's arithmetic, was published in 1630 and is now in the archives of the Dedham Historical Society.[1]

In 1663 a contract with John Swinerton specifies that "The said Mr. Swinerton is to teach such male childeringe as are sent to him to write & read & the use of retmitich as they are capable."[2] In 1667 it was "Agreed with Mr. Samuel Man, to teach the male Children of this towne that shall be sent to him in English Writeing, Grammar, and Arithmeticke."[3] Michael Metcalfe was engaged again, 1679, "to teach all male children that shall be sent to him to Read and wright and cast accounts."[4]

At Plymouth a school was established in 1635, "in which a Mr. Morton taught 'to read, write, and cast accounts.'"[5] At Ipswich a school committee was provided for in 1652 "who shall also consider the best way to make provision for teaching to write and cast accounts."[6] A contract with a teacher at Charlestown in 1671 specifies "that he shall teach to read, write, and cypher."[7] The writer of the History of Hadley makes this comment upon the early schools "The master, with rare exceptions, was a man of collegiate education, and he instructed some in Greek and Latin, but most only in reading, writing, and arithmetic."[8] The first settlement at Newbury was in 1635, and the first school was established in 1639. In 1658 the town paid a fine under the law of 1647 for providing no grammar school. At a town meeting in 1675 "it was voted to have a schoolmaster got to teach to write & read and cypher and teach a grammar schoole."[9] Arithmetic is also specified in contracts dated 1687, 1690, 1691, 1696, 1709–10, 1711–12.[10] The position was held by the same teacher from 1696 to 1709, and probably the other dates represent only the employing of new teachers. In any case we have a fairly continuous record for 25 years, during which time arithmetic was specified in the teacher's contract, and the presumption is that it was taught as early as 1658, certainly as early as 1675.

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1 Op. cit., pp. 15–16.
2 Carlos Slafter: Op. cit., p. 20.
3 Carlos Slafter: Op. cit., p. 32.
4 Carlos Slafter: Op. cit., p. 35.
5 Worcester Society of Antiquity, Collections, Vol. IX, Proceedings, 1888–1890, p. 88.
6 Joseph B. Felt: History of Ipswich, Essex, and Hamilton, pp. 83–84.
7 Richard Frothingham, History of Charlestown, p. 177.
8 Sylvester Judd: History of Hadley, p. 66.
9 John J. Currier: History of Newbury, p. 396.
10 Ibid., pp. 396–400.

On the other hand, arithmetic is not mentioned in the contracts in a number of towns. For example, the town of Dorchester voted in 1639 that, "There shal be a rent of 20[lb] yeerly to be payd to such a schoolmaster as shall undertake to teach english latin and oth[r] tongues, and also writing."[1] With reference to the first teacher, Mr. Waterhouse, the town records contain this:

It is ordered that Mr. Waterhouse shall be dispensed with concerning that Clause of the order * * * where he is bound to teach to write it shal be left to his liberty in that poynt of teaching to write, only to doe what he can conveniently therein.[2]

Later contracts likewise contain no mention of arithmetic.

In Connecticut, arithmetic is not mentioned in the contracts of a number of towns with their teachers, although reading and writing are and frequently also Latin.[3] However, the New Haven Court in 1690, decreed that—

two free schools be established in the colony, one at Hartford and the other at New Haven, where the children may come "after they can first read the Psalter, to teach such reading, writing, arithmetic, the Latin and Greek tongues."[4]

Pennsylvania.—William Penn came to America in October, 1682. In March, 1683, the general assembly passed numerous bills relative to the future welfare of the colony of Pennsylvania. The following provision concerning education was contained therein:

And to the end that poor as well as rich may be instructed in good and commendable learning, which is to be preferred before wealth, be it &c. that all persons in this province and territories thereof, having children, and all the guardians or trustees of orphans, shall cause such to be instructed in reading and writing; so that they may be able to read the Scriptures; and to write by that time they attain to twelve years of age; and that then they be taught some useful trade or skill, that the poor may work to live, and the rich, if they become poor, may not want.[5]

Although arithmetic is not mentioned, it seems to have been recognized as having a legitimate place in the curriculum, for we find that on December 26 of the same year a council held at Philadelphia acted as follows:

The Governor and Provincial Council, having taken into their serious consideration the great necessity there is of a schoolmaster for the instruction and sober education of youth in the town of Philadelphia, sent for Enoch Flower, an inhabitant of the said town, who for twenty years past hath been exercised in that care and employment in England, to whom having communicated their minds, he embraced it upon these following terms: to learn to read English, 4s. by the quarter, to learn to read and write, 6s. by the quarter, to learn to read, write and cast account, 8s. by the quarter; for boarding a scholar, that is to say, diet, washing, lodging and schooling, ten pounds for one whole year.[6]

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[1] William Dana Orcutt: Good Old Dorchester, p. 290.

[2] Ibid., p. 292.

[3] Edward E. Atwater: History of the Colony of New Haven, pp. 262-267. William Chauncey Fowler, History of Durham, Conn., p. 103.

[4] Samuel Orcutt and Ambrose Beardsley: History of the Old Town of Derby, Connecticut, p. 106.

[5] Elsie W. Clews: Op. cit., p. 281.

[6] Elsie W. Clews: Op. cit., pp. 281-282.

Delaware and New Jersey.—The remaining colonies exhibit no striking educational characteristics. Delaware and New Jersey show something of the characteristics of both New York and Pennsylvania. New Jersey was originally included in New York, and later West Jersey was a part of Pennsylvania. Delaware was first settled by the Dutch and Swedes, but later came under the control of Pennsylvania.

In Delaware the design of the "Friends' Public School," now known as the "William Penn Charter School," is set forth in the preamble to the charter as follows:

> Whereas the prosperity and welfare of any people depend, in great measure, upon the good education of youth and their early introduction in the principles of true religion and virtue, and qualifying them to serve their country and themselves by educating them in reading, writing, and learning of languages, and useful arts and sciences suitable to their sex, age and degree, which can not be effected in any manner so well as by erecting public schools for the purposes aforesaid, etc.[1]

The spirit of this and the content of the education outlined is almost identical with the provisions made for education in Pennsylvania. The mention of "useful arts and sciences suitable to their sex" probably means arithmetic for boys.

Southern colonies.—To the south of Pennsylvania, the population was scattered on great plantations and not collected in villages and cities. The education of the masses was almost wholly neglected. The rich employed private tutors. Sometimes instruction was given by the minister or by an indentured servant who possessed education. The aim of this education was usually to prepare for college and did not include instruction in arithmetic.[2]

When the legislatures of the southern colonies registered their attitude on education, arithmetic was usually included in the school curriculum. For example, in 1710 the Legislature of South Carolina passed "An act for the Founding and Erecting of a Free School for the Use of the Inhabitants of South Carolina." It says in part:

> XI. And be it further enacted by the authority aforesaid, that the person to be master of the said school, shall be of the religion of the Church of England, and conform to the same, and shall be capable to teach the learned languages, that is to say, the Latin and Greek tongues, and also the useful parts of the mathematics.
>
> XV. And because it is necessary that a fit person teach the youth of this province to write, and also the principles of vulgar arithmetic and merchants' accounts, Be it therefore enacted by the authority aforesaid, that a fitting person shall be nominated and appointed by the said commissioners, to teach writing, arithmetic, and merchants' accounts.[3]

Two years later there is record of an appointment in which—

> John Douglas shall be and is hereby declared to be Master of a Grammar School in Charleston, for teaching the Greek and Latin languages, and shall choose one usher

[1] Lyman P. Powell: The History of Education in Delaware, p. 30.
[2] F. P. Graves: A History of Education in Modern Times, p. 82.
[3] Elsie W. Clews: Op. cit., pp. 448ff.

to the said school, who is empowered and required to assist the master aforesaid in teaching the languages, reading, English, writing, arithmetic or such other parts of the mathematics as he is capable to teach.[1]

Summary.—This survey of the early schools of the American colonies shows that, whether arithmetic was explicitly mentioned along with reading and writing in the official acts of the colonial governments, as in New York, or was omitted, as in the case of Massachusetts and Pennsylvania, arithmetic was taught in the public school in many towns, probably from the beginning. The activities of trade and commerce, which were centered in these towns, created a demand for arithmetic, and instruction was given in the subject either in the public schools or in private institutions. In these schools arithmetic was primarily a tool of commerce.

In Massachusetts the law of 1647 specified two types of public schools. For towns of 50 householders or more it was ordered that they "appoint one within their town to teach all such children as shall report to him to write and read." For towns of 100 or more families it was ordered that they establish a grammar school, a school of secondary rank, "the master thereof being able to instruct youth so far as they may be fitted for the university." Since arithmetic was not required for college entrance before the middle of the eighteenth century, it was not officially given a place in either type of school. But it was frequently mentioned in teachers' contracts coordinately with reading and writing. Occasionally arithmetic was taught by the master of the grammar school; or an assistant, called an usher, was appointed whose duties included the giving of instruction in writing and arithmetic. In general, when arithmetic was taught in the public schools it was in the elementary rather than the grammar school.

In addition to these types of public schools, there were two types of private schools. One of these was for very young pupils and was known as a dame school. In the dame school the simplest rudiments of arithmetic, such as the addition and multiplication tables, were sometimes taught. The other type of private school, frequently called a writing school, was for the distinct purpose of giving instruction in writing and arithmetic. In case arithmetic was not taught in the grammar school in which a pupil was enrolled, he often attended a writing school half of the day or of evenings. By establishing public writing schools Boston created a "double-headed" school system which persisted well into the nineteenth century. But this practice was not general.

Arithmetic as a science of numbers was taught in some of the colleges, particularly toward the close of this period. After 1750 it had a place in the course of study of many of the academies.

[1] Elsie W. Clews: Op. cit., p. 457.

The growth of arithmetic as a school subject.—In 1789 the teaching of reading, writing, and arithmetic was made obligatory in both Massachusetts and New Hampshire. It is not unreasonable to suppose that these laws simply represent the legalizing of a practice which was already prevalent. Whether this is the case or not, the enactment of these laws shows that arithmetic was then considered necessary to an elementary education and was given a place coordinate with reading and writing. The following records of attendance in the Boston schools indicate the increasing popularity of the writing school, the special school for giving instruction in arithmetic and writing:

	1739.	1768.
North Writing School	280	220
North Grammar School	60	36
South Grammar School	120	115
South Writing School	62	240
Writing School in Queen Street [1]	73	230

In 1745 Yale required arithmetic for entrance. In 1760 Princeton required the candidates "to understand the principal rules of vulgar arithmetic." In 1807 Harvard required—

Candidates for admission into Harvard College shall be examined by the President, Professors and Tutors. No one shall be admitted, unless he be thoroughly acquainted with the Grammar of the Greek and Latin languages, in the various parts thereof, including Prosody, * * * can properly construe and parse Greek and Latin authors, * * * be well instructed in the following rules of arithmetic, namely, Notation, simple and compound Addition, Subtraction, Multiplication, and Division, together with Reduction and the single Rule of Three, * * * have well studied a Compendium of Geography, * * * can translate English into Latin correctly, * * * and have a good moral character, certified in writing by the Preceptor of the Candidate, or some other suitable person.[2]

By 1814, the reference to arithmetic was changed to "and be well instructed in Arithmetic through the Single Rule of Three," and in and after the year 1816, "the whole of Arithmetic."

This recognition of arithmetic in the college entrance requirements necessitated the teaching of arithmetic in the grammar schools.

The establishment of a new type of school, the academy, which included arithmetic in its curriculum from the first, and the subsequent rapid rise of the academy evidences the growing appreciation of arithmetic and other forms of elementary mathematics. In the first academy, established at Philadelphia as the result of the labors of Benjamin Franklin, there were three departments or schools, the Latin, the English, and the mathematical.

The production of arithmetic texts by American authors and the numerous editions of texts by English authors which were published in this country in the latter part of this period also indicate the

[1] D. C. Colesworthy: John Tileston's School, p. 15.
[2] E. E. Brown: The Making of Our Middle Schools, p. 249.

increasing interest in the subject. American arithmetics may be said to date from 1788, the year in which Nicolas Pike published *A New and Complete System of Arithmetic composed for the use of the citizens of the United States.* The publishing of Pike's book seems to have been the signal for the appearance of texts by American authors. By 1800, at least 20 arithmetics[1] by American authors had been published, besides several of not purely arithmetical nature, such as, *Instructor*, 1794; *The Traders Best Companion*, 1795; and an American adaptation of John Gough's *Treatise of Arithmetic*, 1788.

In the 21 years which elapsed between 1800 and the close of this period, arithmetics by American authors appeared with increasing frequency. *The Scholar's Arithmetic*, by Daniel Adams (first published in 1801) had passed through nine editions by 1815. Daboll's *Schoolmaster's Assistant* (first published in 1799) was even more popular. Other American texts had an extended circulation. Numerous editions of Dilworth's *Schoolmaster's Assistant* (first published in England in 1743) were reprinted in this country. A revision of this popular text, by Daniel Hawley, was published in 1803.

In his American Journal of Education, Henry Barnard gives reminiscences by a number of persons who attended school in the last quarter of the eighteenth century.[2] One writing from rural Connecticut says that "arithmetic was hardly taught in day school" but adds that it was taught in evening schools. Only two say that arithmetic was not taught, but they were prepared for college in academies about 1780 and presumably never attended an elementary school. Ten who attended school in rural districts, including the States of Massachusetts, Connecticut, Pennsylvania, New Jersey, and North Carolina, say that arithmetic was taught. Most of them mention it coordinately with reading and writing. Three make no mention of arithmetic, and four studied arithmetic in cities.

The appearance of arithmetic in the college entrance requirements, the activity of American authors in writing texts, and the direct tes-

[1] This number includes arithmetics by the following authors: Isaac Greenwood (1729), Benjamin Dearborn (1782), Alexander McDonald (1785), Nicolas Pike (1788), Thomas Sarjeant (1788), Consider and John Sterry (1790), John Vinall (1792), Benjamin Workman (1793), Joseph Chaplin (1795), Daniel Fenning (1795), Erastus Root (1796), James Noyes (1797), Chauncey Lee (1797), William Milns (1797) David Kendall (1797), Peter Tharp (1798), Zachariah Jess (1798), Ezekiel Little (1799), Nathan Daboll (1799), David Cook (1800). In his American Bibliography, Evans accredits a text to Jonathan Burnham in 1748. It has not been possible to verify this.

[2] The names, the States in which they attended school, and the years attended are given in the Analytical Index to Barnard's American Journal of Education as follows (the figures following the dates refer to the volume and page on which the reminiscence is given): Allen, Mrs. L. L., Massachusetts, 1796–1808, 30:581; Buckingham, J. T., Connecticut, 1783, 13:129; Bushnell, Rev. H., Connecticut, 1800, 13:142; Caldwell, C., North Carolina, 1790, 16:109; Channing, Rev. W. E., Rhode Island, 1780, 17:189; Darlington, W., Pennsylvania, 1795, 13:741; Davis, John, Virginia, 1800, 13:748; Day, Rev. J., Connecticut, 1780, 16:125; Everett, E., Massachusetts, 1800, 13:747; Goodrich, S. G., Connecticut, 1800, 13:134; Hall, W., Massachusetts, 1785, 16:127; Hedges, N., New Jersey, 1800, 16:738; Humphrey, Rev. H., Connecticut, 1790, 13:125; Nott, Rev. E., Connecticut, 1780, 13:132; Oliver, H. K., Massachusetts, 1805, 26:209; Quincy, J., Massachusetts, 1778, 13:740; Seton, S. W., New York, 1795, 17:555; Town, S., Massachusetts, 1785, 13:737; Webster, D., New Hampshire, 1790, 27:282; Webster, M., Connecticut, 1770, 26:197, 13:123.

timony of these persons show that, by 1800, arithmetic was generally taught in the schools, even in the country districts.

The increasing recognition of arithmetic as an essential school subject is but one element of a larger change which culminated in the nineteenth century in the complete secularization of public schools in this country. The control of education passed from the church to the state, and instead of education primarily for teaching the catechism and church doctrines the purpose of education came to be a preparation of children for the secular activities of life. In the period from the close of the Revolution to 1821 arithmetic grew rapidly in importance as a school subject, and in later chapters it will be shown that it was given a place of prime importance in the secularized concept of education.

The aim of instruction in arithmetic.—The aim of arithmetical instruction in this period was not well defined. In a general way the practical needs of trade and commerce were to be satisfied, and this was the principal aim. The authors of the texts used clearly thought of arithmetic primarily as a commercial subject. James Hodder says in the preface to his arithmetic, or, *That Necessary Art Made Easy* (first published 1661 and widely used in the colonies): "And now for the better compleating of youth, as to clerkship and trades, I am induc'd to publish this small treatise of Arithmetic." The title of Greenwood's book, *Arithmetick Vulgar and Decimal: with the Application thereof, to a variety of Cases in Trade, and Commerce*, indicates a similar recognition of the practical aim. Daboll says in the preface to Daboll's *Schoolmaster's Assistant* (first published 1799): "The design of this work is to furnish the schools of the United States with a methodical and comprehensive system of *Practical Arithmetic*." A ciphering book prepared in Boston, in 1809, has the following title: *Practical Arithmetic composing all the Rules necessary for transacting Business.*

The immediate end sought, which also represents the standard of instruction, was a knowledge of the rules and their application. We shall show in another place that the pupil was expected to learn the rule and then to apply it to a very few examples or problems. No opportunity was given for drill upon the application of the rule, even in the case of the fundamental operations. Skill and facility were not expected nor attempted.

Dilworth's *Schoolmaster's Assistant* contains only 9 examples for drill on addition, a like number on subtraction, and a somewhat greater number on multiplication and division. Pike's arithmetic, which is an elaborate text of 512 pages,[1] contains only 9 examples for drill on addition and 9 in subtraction. Subtraction is disposed of

[1] Only 408 pages are devoted to arithmetic. See Appendix for table of contents.

within a single page. Adams's *Scholar's Arithmetic* contains 10 examples for drill on addition and 9 on subtraction.

Reminiscences and records of the schools of this period indicate that the pupil actually solved even a less number of drill examples than were given in the texts. The compiler of this report has in his possession a copy of Adams's *Scholar's Arithmetic* in which blank places are left for the solution of the problems. Only 5 of the 10 problems in addition are solved and only 2 of the 9 in subtraction. The examination of other texts and of ciphering books written in this period reveals about the same amount of drill work.

William B. Fowle relates the following which is probably typical:

> No boy had a printed arithmetic, but every other day a sum or two was set in each manuscript, to be ciphered on the slate, shown up, and if right, copied into the manuscript. Two sums were all that were allowed in subtraction, and this number was probably as many as the good man could set for each boy. This ciphering occupied two hours, or rather consumed two, and the other hour was employed in writing one page in a copy book. Once, when I had done my two sums in subtraction, and set them in my book, and been idle an hour, I ventured to go to the master's desk and ask him to be so good as to set me another sum. His amazement at my audacity was equal to that of the almshouse steward when the half-starved Oliver Twist "asked for more." He looked at me, twitched my manuscript toward him, and said, gutturally "Eh, you gnarly wretch, you are never satisfied." I had never made such a request before, nor did I ever make another afterwards.[1]

Furthermore, there was very little attempt made to develop ability to apply the rules except to problems explicitly falling under given rules. If a problem appeared which could not be readily classified as coming under some known rule, both pupil and teacher were usually at a loss to know how to proceed. Occasionally there was a pupil who developed some real ability to reason out problems and to control unfamiliar arithmetical situations. However, this was the exception and happened not in response to a conscious attempt on the part of the teachers, but rather in spite of the system.

[1] The Teacher's Institute, or Familiar Hints to Young Teachers, p. 61.

Chapter II.

THE SUBJECT MATTER OF ARITHMETIC BEFORE 1821.

With few exceptions the texts in use in the United States before 1800 were of English authorship. Copies of these texts were imported, and editions of the popular ones were printed in this country. The "first purely arithmetical work published in the United States" was an edition of Hodder's arithmetic, printed in Boston in 1719 by J. Franklin.[1] Editions of the texts by Cocker, Wingate, Bonnycastle, Gough, and Dilworth were printed in this country. In settlements other than English, notably New York and Pennsylvania, arithmetics written by their countrymen were used.

The Schoolmaster's Assistant, by Thomas Dilworth, originally published in 1743, was used very extensively in this country, almost exclusively prior to 1800. Numerous editions were printed in this country, and after the adoption of a Federal money it was revised to meet the commercial needs. A revised edition was published by Daniel Hawley in 1802 with the title of *Federal Calculator*. This revision had passed through five editions by 1817. Revised editions of this revision, by William Stoddard, were published in 1817 and in 1832.

On page 14 there is printed a list of the American authors of arithmetics published by 1800. Few of these texts were used extensively. The first arithmetic by an American author, *Arithmetick, Vulgar and Decimal: with the applications thereof, to a Variety of Cases in Trade and Commerce*, by Isaac Greenwood, 1729, found no place in the schools and was soon forgotten. In fact all of the texts prior to the one by Nicolas Pike in 1788 were so little known that his text was considered by some to be the first by an American author. Pike seems to have held this opinion himself. Although not the first text, this book, which was entitled *A New and Complete System of Arithmetic*, marked the beginning of arithmetic adapted to the needs of the United States. It comprised 512 pages, of which the first 408 are devoted to arithmetic and closely related topics and problems. There follow 4 pages of "plain" geometry, 11 pages of "plain" trigonometry, 45 pages of mensuration of superficies and solids, 33 pages of "an introduction to algebra, designed for the use of academies," and 10 pages of an introduction to conic sections.[2]

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[1] Evans's American Bibliography, Vol. I, p. 272.

[2] The complete table of contents is given in the Appendix, p. 152.

Pike's arithmetic is an elaborate treatise and not a text for the use of young pupils. It represents the maximal content of arithmetic in this period. The book sold for $2.50, which placed it out of the reach of many pupils. It was used primarily in academies and colleges and yet it had a considerable circulation. A second edition was printed in 1797, a third in 1808, a fourth in 1822, and a fifth in 1832. An abridged edition was published in 1793, and a second one, prepared by Dudley Leavitt, appeared in 1826.

Following 1788, texts by American authors appeared with increasing frequency. *The American Tutor's Assistant*, by Zachariah Jess, 1798; *The Schoolmaster's Assistant*, by Nathan Daboll, 1799; *A New System of Mercantile Arithmetic*, by Michael Walsh, 1800; *Scholar's Arithmetic*, by Daniel Adams, 1801; and *Scholar's Arithmetic*, by Jacob Willetts, 1817, were widely used.

Of these texts, Daboll's *Schoolmaster's Assistant* seems to have been most popular. An edition "improved and enlarged," was published as late as 1839. Adams's *Scholar's Arithmetic* had passed through 9 editions, and 40,000 copies had been sold when it was revised in 1815. An edition was published in 1822. Jacob Willetts's *Scholar's Arithmetic* passed "through more than 50 editions in a few years." A revised edition was published in 1849. A third revised edition of 20,000 copies of Walsh's *Mercantile Arithmetic* was printed in 1807. An edition was published as late as 1826.

The content of the texts.—Since Dilworth's *Schoolmaster's Assistant* was the first text in arithmetic to attain an extended circulation in this country, it will be used as a basis for an exposition of the content of the texts of this period. Reference will be made to features of other popular texts which were significant.

The theory of arithmetic.—Theoretical arithmetic was recognized in the definitions of arithmetic which were given in these early texts. The space given to arithmetical theory varied. Dilworth's text is primarily a practical arithmetic and he gives very little in the way of demonstrating "the reason of practical operations," and he has nothing to say about "the nature and quality of numbers." Pike attempts to treat comprehensively both theoretical and practical arithmetic. The spirit of the mathematician who is interested in the theory of numbers and operations pervades the whole book. In footnotes he demonstrates the operations. Under the head of "Vulgar Fractions" he defines prime number, composite number, and perfect number, and gives 10 perfect numbers which he states are "all which are, at present known." The other texts of the period show much less emphasis upon arithmetical theory. Often considerable space was given to a "demonstration" of the rules, but these demonstrations were usually explanations of the application of a rule to a particular problem or example.

Definitions.—The definitions of number, fraction, addition, etc., were usually given in an abstract form, with no reference to the concrete situations which required the arithmetical concept or operation. For example, addition was defined as "putting together two or more numbers or sums, to make them one total, or whole sum." In the case of business rules, an attempt was made to indicate the sort of situation which called for the particular rule. But the practical situation itself was not described except in the problems. There was usually no attempt to build up a logical system of definitions.

Notation and numeration.—Dilworth made this topic, which he styles, "Notation," the first in the text after some preliminary definitions. Numeration consisted of rules for reading numbers, and they are given for reading numbers up to 9 digits. Pike's rule extends to sextillion, 42 digits, and in a note to duodecillion, 78 digits. The periods are of 6 digits each. Daboll also used 6 digits to a period, and he gives four such periods. "Notation of numbers by Latin letters" is mentioned, but not given by Dilworth. Wingate gives Roman numerals and prefers IIII to IV, VIIII to IX, etc., and IIX is given with VIII for eight. Pike gives Roman notation, but Daboll and many other authors omit it.

The fundamental operation for integers.—These operations were given in the serial order, addition, subtraction, multiplication, and division. Sometimes this order was interrupted to give the tables of denominate numbers after addition. This is the case in Dilworth's text. In addition he gives the rule for placing the numbers to be added and recommends proving by adding in reverse order. He does not mention "carrying" and solves out no examples. Nine abstract examples are followed by 15 pages of "compound" addition. The rule for subtraction is given, but otherwise the presentation is similar to that of addition. In multiplication, the tables are given from 3 to 12 inclusive, except the tens. The process of multiplication is given in five cases: First, when the multiplier is 12 or less; second, when the multiplier "consists of more figures than one"; third, when the factors "have cyphers at the ends"; fourth, when the multiplier has cyphers "between the significant figures"; fifth, when the multiplier may be resolved into two factors, each being less than 10. Short division is disposed of with no rule and only 12 examples. Long division is taken up in three cases, with a rule for each: First, any divisor; second, when there are cyphers at the end of the divisor; third, when the divisor "is such a number" that it is the product of "any two figures." In no case is an example worked out as a model or the rule explained. Besides each operation being applied to "compound numbers," there is also a list of practical problems for each rule.

In other texts the fundamental operations are presented in a more simplified form. Cocker, in general, explains a process before he applies it to a particular example. Hodder carefully explains an example, even in addition, before he states the rule. Pike and Daboll give addition and subtraction tables. Most authors give the table of Pythagoras.[1] Pike "demonstrates" the rule for multiplication and division. Cocker and Hodder attempt to add to the understanding of multiplication and division by telling of the situations which require the operations. Hodder speaks of multiplication as being equal to many additions. Daboll says "division is a concise way of performing several subtractions." The forms of the operations are essentially the same as our present forms with one or two exceptions in the older English texts.

In addition to the five cases of multiplication given by Dilworth, Pike recognizes the seven following cases: First, to multiply by 10, 100, 1,000, etc.; second, "to multiply by 99, 999, etc., in one line"; third, "to multiply by 13, 14, 15, etc., to 19, inclusively, at one multiplication"; fourth, "to multiply by 111, 112, 113, to 119, so as to have the product in one line;" fifth, "to multiply by 101, 102, 103, etc., to 109, so as to have the product in one line;" sixth, "to multiply by 21, 31, 41, etc., to 91, in one line;" seventh, "to multiply by 22, 23, 24, etc., to 29, so as to have the product in one line." In addition to these 12 cases a general rule is given for multiplying "any number, viz, whole or decimal, by any number, giving only the product." Detailed specific rules are given for each case; for some cases two such rules are given. But there is a marked tendency in the texts after Pike's in the direction of fewer cases. Daboll recognizes only five cases and Adams gives besides the general rule only a section to "contractions and varieties in multiplication."

A knowledge of the addition and subtraction facts seems to have been taken for granted. Some of the texts do not give an addition or subtraction table. The multiplication and division tables are usually given and were to be memorized. Adams says under multiplication, "Before any progress can be made in this rule, the following table must be committed perfectly to memory." There are no exercises to be solved orally, and there is no provision for drill upon the number facts contained in the tables.

Common, or vulgar fractions.—Dilworth devotes Part II of his text to vulgar fractions (see Appendix). Following the definition of a fraction as "any two numbers placed thus, $\frac{4}{5}$," and the definition of terms and the "sorts of vulgar fractions," reduction of fractions is given in 12 cases. They are: (1) Reduction to common denominator; (2) reduction to lowest terms; (3) and (4) reduction of "mixt" number, to improper fraction and reverse; (5) reduction of compound fraction to a single fraction; (6) to reduce a fraction of one

[1] A form of the multiplication table.

denomination to a fraction of another, but greater; (7) to reduce a fraction of one denomination to a fraction of another but less; (8) to "reduce vulgar fractions from one denomination to another of the same value, having the numerator of the required fraction given"; (9) the same except the denominator of the required fraction is given; (10) to reduce "a mixed fraction to a single one"; (11) to "find the proper quantity of a fraction in the known parts of an integer"; (12) "to reduce any given quantity to the fraction of any greater denomination of the same kind." The operations of addition, subtraction, multiplication, and division for fractions are then disposed of within three scant pages. Two pages devoted to the single rule of three direct, single rule of three inverse, and double rule of three for vulgar fractions complete Part II. For each of the four operations a specific rule is given, e. g., the rule for multiplication is, "Multiply all the given numerators for a new numerator, and all the denominators for a new denominator."

For reducing a fraction to its lowest terms, Dilworth gives only the Euclidean process. In general the other authors give the rule, "Divide the terms of the given fraction by any number which will divide them without remainder, and the quotients, again, in the same manner; and so on till it appears that there is no number greater than 1 which will divide them." Pike and Daboll give both methods. Dilworth's rule for reducing fractions to a common denominator is: "1. Multiply each numerator into all the denominators but its own for a new numerator. 2. Multiply all the denominators for a new denominator." The least common denominator is not mentioned, although it would be very useful in the examples he gives. Pike and Daboll give in addition the method for reducing to a least common denominator.

Dilworth does not solve an example or illustrate a rule. Cocker and Hodder and the later authors, in general, solve out one example under a rule and usually carefully explain the operation. Wingate suggests cancellation as a short method in multiplication of fractions. Daboll also does this. Pike gives three cases under multiplication.

The contrast in the position and space given to common fractions is interesting. Hodder and Pike place them immediately following denominate numbers and reduction. Daboll gives three cases of reduction of fractions immediately following denominate numbers, but the real treatment of the topic comes nearly 100 pages later in the text. Adams finishes with fractions with a scant page devoted to explaining the meaning of a vulgar fraction and closes by saying: "The arithmetic of vulgar fractions is tedious and even intricate to beginners. We shall not therefore enter into any further consideration of them here."

Dilworth and Daboll make no attempt to explain the meaning of a fraction. They just tell what the symbol is and how it is to be operated upon. Adams gives two illustrations to explain the meaning of a fraction. The examples are abstract, the nearest approach to a practical problem being in such as: "Add $\frac{3}{4}$ of a yard, $\frac{1}{2}$ of a foot, and $\frac{5}{8}$ of a mile together." Factoring, highest common divisor, and least common multiple are not mentioned by Dilworth. Pike gives them as the first topics under the head of fractions.

Vulgar fractions were even omitted in a few texts. Chauncey Lee in *The American Accountant*, 1797, explains his reason for omitting them as follows:

> As the use of vulgar fractions may be advantageously superseded by that of decimals, they are viewed as an unnecessary branch of common school education and therefore omitted in this compendium.

Decimal fractions.—Part III of Dilworth's *Schoolmaster's Assistant*, which bears the title, "Of Decimal Fractions," includes much subject matter which is not commonly included under this head. Besides notation, reduction, addition, subtraction, multiplication, and division for decimals, the section contains evolution, the rule of three, interest, discount, equation of payments, and a number of other applications of percentage. (See Appendix.) The four operations are presented very briefly and entirely abstractly. Reduction includes such examples as, "Reduce 76 yards to a decimal of a mile," and the reverse exercise.

The place occupied by decimal fractions in this text is significant of the esteem in which they were held. As compared with common fractions, the rule of three, interest, partnership, and other topics, decimal fractions were new. The elementary arithmetical processes, with the exception of decimal fractions and logarithms, were matured by the close of the sixteenth century. Simon Stevin gave the first systematic treatment of decimal fractions in 1585, and their application to practical arithmetic was a contribution of the seventeenth century. Coming thus after methods for the calculations of business had been worked out, which were moderately satisfactory, decimal fractions and the methods of calculation which they make possible were incorporated in the texts only very slowly. Hodder, 1661, does not mention them in his table of contents, but approaches them in a chapter on profit and loss. Dilworth, as we have seen, treats all of the more common problems of business before he mentions decimal fractions. This shows that a need for them was not keenly felt.

The establishment of a Federal money, 1786, increased the usefulness of decimal fractions and marked the beginning of their increased importance as a topic of arithmetic in the United States. Pike, who gives a brief account of Federal money immediately after decimal fractions, places them early in his text. Daboll places Federal money immediately after addition of integers, but the position and treatment of decimal fractions is essentially the same as in Pike's text. Adams follows the order of Pike, but gives a less elaborate treatment.

Denominate numbers.—Weights and measures were not standardized, and we find a lack of uniformity in the tables of denominate numbers. Dilworth gives the tables of English money, Troy weight, avoirdupois weight, apothecaries' weight, time, and motion (circular measure), in essentially the form we know them to-day. Other systems of measures are given in a form which is only partially like that in our arithmetics to-day, and there are some which have disappeared from our texts. Because of their value in showing a phase of the development of arithmetic, we give the last two classes of tables below:

LONG MEASURE.

3 Barley Corns, or B. C., make	1 Inch, In.
4 Inches	1 Hand, hd.
12 Inches	1 Foot, ft.
3 Feet	1 Yard, yd.
6 Feet	1 Fathom, Fa.
5 Yards and a half	1 Rod, Pole, or Perch, Po.
40 Poles	1 Furlong, Fu.
8 Furlongs	1 Mile, Mi.
3 Miles	1 League, L.
60 Miles	1 Degree, Deg.

CLOTH MEASURE.

2 Inches, or in., and a quarter make	1 Nail, N.
4 Nails	1 Qr. of a Yard, qr.
4 Quarters	1 Yard, yd.
3 Quarters of a yard	1 Flemish Ell, F. E.
5 Quarters of a yard	1 English Ell, E.

LAND MEASURE.

9 Square Feet, or Ft	1 Yard, Y.
30 Yards and a Quarter	1 Pole, Po.
40 Poles in length and 1 in breadth	1 Rood, R.
4 Roods	1 Acre, A.

LIQUID MEASURE WINE MEASURE.

2 Pints, or pts., make	1 Quart, qt.
4 Quarts	1 Gallon, gal.
10 Gallons	1 Anchor of Brandy or Rum, An.
18 Gallons	1 Runlet, R.
31½ Gallons	1 Barrel, Bar.
42 Gallons	1 Tierce, Tier.
63 Gallons	1 Hogshead, hhd.
84 Gallons	1 Puncheon, Pu.
2 Hogsheads	1 Pipe or Butt, P.
2 Pipes or 4 Hogsheads	1 Tun, T.

WINCHESTER MEASURE.

2 Pints, or pts., make	1 Quart, qt.
4 Quarts	1 Gallon, gal.
8 Gallons	1 Firkin of Ale, Fir.
9 Gallons	1 Firkin of Beer, Fir.
2 Firkins	1 Kilderkin, Kil.
4 Firkins	1 Barrel, Bar.
1 Barrel and a half, or 54 Gall	1 Hogshead of Beer, hhd.

DRY MEASURE.

2 Pints, or pts., make	1 Quart, qt.
2 Quarts	1 Pottle, Pot.
2 Pottles	1 Gallon.
2 Gallons	1 Peck.
4 Pecks	1 Bushel, Bush.
8 Bushels	1 Quarter of Corn.

A supplementary table to avoirdupois weight is also of interest:

	Lbs.
A firkin of butter is	56
——— of soap is	64
A barrel of pot ash is	200
——— anchovies is	30
——— candles is	120
——— figs is from 98 lbs. to 2 cwt. 3 qrs	
——— soap is	256
——— butter is	224
——— gunpowder is	112
——— raisins is	112
A double barrel of anchovies	60
A puncheon of prunes is 10 cwt. or	120
A fother of lead is	120
A stone of iron or shot is 19 cwt. 2 qr.	
——— butchers is	8
A gallon of train oil is	7½
A faggot of steel is	120
A burden of gad steel or 9 score	180
A quintal of fish in Newfoundland is	100
A stone of glass is	5
A seam of glass is 24 stone.	
——— of cheese or butter is	120
A clove or half stone is	8
A wey in Suffolk is 32 cloves, or	256
——— Essex is 42 cloves.	
——— of wool is	336
A clove is	7
A stone is	14
A tod is	28
A wey is 6 tod and 1 stone, or	182
A sack is 2 weys, or	364
A last is 12 sacks, or	4,368

In addition, in the section on exchange, the tables for the money of a number of foreign countries and even cities are given: Spain, Italy, Venice in Italy, France, Portugal, Florence in Italy, Frankfort in Germany, Antwerp, Brussels, Amsterdam, Rotterdam, Hamburg in Germany, British Dominions in America, the West Indies, Ireland, Denmark, and Stockholm in Sweden.

Pike adds to cloth measure:

6 quarters of a yard make 1 ell—French.
4 quarters, 1 inch and one fifth, make 1 ell—Scotch.
3 quarters and two-thirds make 1 Spanish var.

In long measure, he omits the denomination of hand and adds the surveyors' measure. Square measure is increased by the denominations of square inch, square foot, and square mile, and ale or beer measure by the denominations puncheon and butt. The table of dry measure is as follows:

2 pints make	1 quart.
2 quarts	1 pottle.
2 pottles	1 gallon.
2 gallons	1 peck.
4 pecks	1 bushel.
2 bushels	1 strike.
2 strikes	1 coom.
2 cooms	1 quarter.
4 quarters	1 chaldrom.
4½ quarters	1 chaldrom in London.
5 quarters	1 wey.
2 weys	1 last.

Solid (cubic) measure, which Dilworth does not give, is given thus by Pike:

1,728 inches make	1 foot.
27 feet	1 yard.
40 feet of round timber, or 50 ft. of hewn timber	1 ton or load.
128 solid feet, i. e., 8 in length, 4 in breadth, 4 in height	1 cord of wood.

The table of Federal money which was established in 1786 is given by Pike under a section heading, "Decimal tables of coin, weight, and measure." The decimal tables of weight and measure was an attempt to decimalize the tables in common use, though the advantage of the form which he gives is not evident.

In Daboll's text the great majority of the problems are stated in terms of the money of the United States. This is true also in Adams's *Scholar's Arithmetic*. But Federal money did not become generally used until considerably later than 1800. In 1815 Adams deplores the use of English money and "to shew the great advantage which is gained by reckoning in Federal money" he contrasts "the two modes of account, and in separate columns on the same page," places the same questions "in Old Lawful and in Federal Money."

The simplicity of the decimal system, upon which the Federal money was based, was very soon evident and stood out in contrast to the haphazard basis of the other systems of measure. Chauncey Lee, in 1795, commenting upon "our tables of weight and measure" points out that they "are as illy contrived for ease of calculation as can well be imagined." And later he says:

I am persuaded that experience will soon evince the expediency, if not the absolute necessity of Federalizing all the tables of weights and measures and other mixed quantities, which have an immediate relation to commerce, upon a decimal scale.

After showing the inconvenience of vulgar fractions for the purposes of calculation, he says:

> This inconvenience will ever continue to operate in a greater or less degree until this *vulgar* evil is plucked up by the roots—all these surd, untoward fractional numbers banished from practice, and the several denominations in all commercial tables of mixed quantities conformed to our Federal money and established upon a decimal scale. To accomplish this is a task too great for any individual in a republican government. It requires the arm of Congress to effect it.

He follows this with Federalized tables for avoirdupois weight, troy weight, liquid measure, dry measure, cloth measure, apothecaries' weight, and board measure. His plan involves keeping at least one unit in each table the same except in the case of troy weight. The following table illustrates the plan:

FEDERAL AVOIRDUPOIS

10 drams make	1 ounce.
10 ounces	1 pound.
100 pounds	1 hundredweight.
10 hundreds	1 thousand.

The plan was not adopted, and there is no trace of it in the arithmetics of Daboll and Adams, which appeared a few years later.

The *American Accomptant* is interesting historically also because it is the first arithmetic in which the dollar mark ($) appears. The mark is in the form //$. There is also a mark for dimes (/x), a mark for cents (//), and a mark for mills (/). But these are scarcely used in the text.[1] Daboll gives our present dollar mark, but uses also the abbreviation "dols." He writes both 127 dols., 19 cents, and $381, 72 cents.

Daboll considers "Federal coin" so "nearly allied to whole numbers, and so absolutely necessary to be understood by everyone" that he introduces it immediately following whole numbers. Adams places it after decimal fractions and 45 pages after table of English money.

The four fundamental operations were usually repeated for denominate numbers under the head of "Compound Addition," "Compound Subtraction," etc. Dilworth divides his treatment of each of the operations into two parts, "simple" and "compound." Pike and Daboll give the operations for "compound" numbers after all operations have been given for "simple" numbers. There are no special rules in Dilworth's text for the operations with "compound" numbers, but other authors usually give specific rules. Pike recognizes as many as eight cases of "Compound Multiplication." Reduction, ascending and descending, was an important topic in the texts.

[1] For a discussion of the origin of the dollar mark, see F. Cajori: The Evolution of the Dollar Mark, Popular Science Monthly, vol. 81, p. 521.

It occupies 10 pages in Dilworth's text, which marks it as one of the most important topics—addition, practice, and exchange being the only ones which are given more space.

Rule of three.—There are three cases of the rule of three which Pike defines as follows:

The Single Rule of Three Direct teacheth, by having three numbers given, to find a fourth, that shall have the same proportion to the third, as the second hath to the first.

The Single Rule of Three Inverse teacheth, by having three numbers given, to find a fourth, which shall have the same proportion to the second, as the first has to the third.

The Double Rule of Three teacheth to resolve such questions as require two, or more, statings by simple proportion; and that, whether direct or inverse. It is composed (commonly) of 5 numbers to find a sixth, which if the proportion be direct, must bear such proportion to the fourth and fifth as the third bears to the first and second: but if inverse, the sixth number must bear such proportion to the fourth and fifth as the first bears to the second and third.

For centuries this rule was the basis of the rules for solving most of the problems arising in business. Its application was made so universal that it was often spoken of as "The Golden Rule" of arithmetic. We shall describe the three forms of the rule and then illustrate the variety of practical situations to which it was applied in the arithmetics of this period.

The rule given for the case of direct proportion was to pick out "the number that asks the question" for the third term, take the one of the "same name or quality" for the first term, and the remaining one which has the same name or quality as the required answer is the second term. The solution is then accomplished by multiplying the second and third terms together and dividing by the first, the quotient being the answer.

The problems under this rule were of the type: "If 6 lbs. of sugar cost 9s., what will 30 lbs. cost at the same rate?" This type of problem was often complicated, as when the first and third terms were not of the same denomination, or when a term was expressed in more than one denomination. Pike recognizes seven cases of these complications for which he gives special directions.

Problems requiring the rule of three inverse are to be distinguished from those belonging to the direct case—

by an attentive consideration of the sense and tenor of the question proposed: for if thereby it appears that when the third term of the stating is less than the first, the answer must be less than the second, or when the third is greater than the first, the answer must be greater than the second, then the proportion is direct; but, if the third be less than the first, and yet the sense of the question requires the fourth to be greater than the second, or if the third being greater than the first, the answer must be less than the second, the proportion is inverse.

The required answer is then obtained by multiplying the first and second terms together and dividing the product by the third. Such problems as the following are placed under this rule:

"What length of board 7¼ inches wide will make a square foot?" "How many yards of carpet, 2¾ feet wide, will cover a floor which is 18 feet long and 16 feet wide?"

Under the double rule of three we find such problems as:

If £100 gain £6 in a year, what will £400 gain in 9 months? If 6 men build a wall 20 feet long, 6 feet high, and 4 feet thick in 16 days, in what time will 24 men build one 200 feet long, 3 feet high, and 6 feet thick?

These are to be solved by two or more successive applications of the single rule of three or by a special rule which is given.

Such problems as, "If 40 lb. at New York make 48 lb. at Antwerp, and 30 lb. at Antwerp make 36 lb. at Leghorn, how many lb. at New York are equal to 144 lb. at Leghorn?" were placed under the separate head of "Conjoined Proportion."[1] There were two cases depending upon whether the question demanded how many of the first measure were equivalent to a given number of the last, as in the problem above, or how many of the last measure were equivalent to a given number of the first. For each case a rule was given.

Practice.—Fourteen pages of Dilworth's text is devoted to "Practice," and judged from this point of view this is the most important topic. It is defined by Pike as a—

contraction of the Rule of Three Direct, when the first term happens to be a unit, or one; and has its name from its daily use among merchants and tradesmen, being an easy and concise method of working most questions which occur in trade and business.

As a preliminary, a table of aliquot, or even, parts of money is to be learned. Pike adds a table of aliquot parts of weight and a table of discount. Practice itself is divided by Dilworth into 10 cases, by Pike into 28, and by Daboll into 6, who explains that "reckoning in Federal money will render this rule almost useless."

The cases given by Pike are:

When the price of 1 yd., lb., etc., is an even part of one shilling.
When the price is pence, and no even part of a shilling.
When the price is pence or farthings, and an even part of a pound.
When the price is between one and two shillings.
When the price is any even number of shillings under 40.
When the price wants an even part of 2s.
When the price is between 2s. and 3s.
When there are pence in the price which are an even part of a shilling, besides an even number of shillings under 20.
When the price is any odd number of shillings under 40.
When the price is an even part of a pound.
When the price wants an even part of a pound.
When the price is shillings, pence, and farthings and not an even part of a pound.
When the price of a yard, lb., etc., is pounds, shillings, and pence.

[1] This method of solving such problems was formerly known as the "Chain Rule." (See Jackson: The Educational Significance of Sixteenth Century Arithmetic, pp. 148–49.)

When the quantity is any number less than 1,000, and the price not more than 12d. per yard, etc.

When the price is such a number of shillings and pence, as, when reduced into pence, may be produced by any two numbers in the multiplication table, and when the quantity does not exceed 1,000.

When the quantity is 240.

When the quantity is not less than 228, nor more than 252.

When the quantity is 480.

When the quantity is 160.

When the quantity is 120.

When the quantity is 80 yards, etc.

When the quantity is 60 yards, etc.

When the quantity is 180.

When the price of one hundredweight is of several denominations, and the quantity likewise.

When the price is at any of the rates in the second Practice-Table of aliquot parts.

When the price is any even number of shillings, if it be required to know what quantity of any thing may be bought for so much money.

To find the discount of any invoice, of bill of parcels, at any rate per cent.

To find the value of goods sold by particular quantities.

Although the authors insist that "Practice" is "a contraction of the Rule of Three," there is no trace of the rule of three in many of the specific rules which are given for the numerous cases. For example, the rule for case 5 mentioned above is: "Multiply the given quantity by half the price, and double the first figure of the product for shillings; the rest of the product will be pounds." There is no effort to give a reason for the rules. The pupil is expected to accept them on faith.

Barter.—Barter was a topic which included such problems as, "How much rice at 28s. per cwt. must be bartered for 3½ cwt. of raisins at 5d. per lb.?" Such a problem was solved by the rule of three.

Fellowship.—The topic of fellowship, later called partnership, was treated in the texts of this period as an application of the rule of three. The rule for single fellowship, i. e., fellowship with equal time, is, "as the whole stock is to the whole gain or loss, so is each man's particular stock to his particular share of the gain or loss." Problems in double fellowship, or fellowship with time, are to be solved by a similar rule.

Alligation.—Such problems as the following are given under the head of alligation medial: "A farmer mingled 19 bushels of wheat at 6s. per bushel, and 40 bushels of rye at 4s. per bushel, and 12 bushels of barley at 3s. per bushel, together; I demand what a bushel of this mixture is worth?" The rule is:

As the whole composition is to its total value, so is any part of the composition to its mean price.

Alligation alternate is defined as "when the rates of several things are given to find such quantities of them as are necessary to make a mixture which may bear a certain rate." There are three cases which are illustrated by the following problems:

A grocer would mix three sorts of sugar together, viz, one sort at 10d. per lb., another at 7d., and another at 6d.; how much of each sort must he take that the whole mixture may be sold for 8d. per lb.?

A man being determined to mix 10 bushels of wheat at 4s. per bushel with rye at 3s., with barley at 2s., and with oats at 1s. per bushel, I demand how much rye, barley, and oats must be mixed with the 10 bushels of wheat that the whole may be sold for 28d. per bushel?

A grocer hath 4 sorts of sugar, viz, at 8d. per lb., at 6d. per lb., at 4d. per lb., and at 2d. per lb., and he would have a composition of an cwt. worth 5d. per lb. I demand how much of each sort he must take?

The rule for solving such a problem as the first is given by Pike as:

"1. Place the several prices of the simples [samples], being reduced to one denomination, in a column under each other, the least uppermost, and so gradually downward, as they increase, with a line of connection at the left hand, and the mean price at the left hand of all.

2. Connect, with a continued line, the price of each simple, or ingredient, which is less than that of the compound, with one or any number of those, which are greater than the compound, add each greater rate or price with one, or any number, of the less.

3. Place the difference, between the mean price (or mixture-rate) and that of each of the simples, opposite to the rates with which they are connected.

4. Then, if only one difference stand against any rate, it will be the quantity belonging to that rate, but if there be several, their sum will be the quantity.

To solve problems such as the second and third above, the rule of three is applied in addition to the above rule. It is to be noted that such problems are in general indeterminate. The answer obtained in any case depends upon the manner in which the second step of the solution is performed. Also, as we have noted before, no attempt is made to give any reason why such a procedure should give the required result.

Position.—Position is the title of the topic which contains such problems as:

Two men, A and B, having found a bag of money, disputed who should have it; A said the half, third, and fourth of the money made 130 L and if B could tell how much was in it, he should have it all, otherwise he should have nothing; I demand how much was in the bag?

A, B, and C would divide 100 L between them, so that B may have 3 L more than A, and C 4 L more than B; I demand how much each man must have.

Such problems as the first are solved by supposing a number and then applying the rule of three. In problems such as the second one above, two numbers are to be supposed and the errors manipulated according to a mechanical rule to determine the corrections which must be made in the supposed numbers.

Exchange.—Under the head of denominate numbers the lack of common standards of weights and measures was noted. This was particularly true of the medium of exchange. Each country, and sometimes even an important commercial city, had its own medium of exchange. (See p. 24.) This situation gave rise to many problems of exchange.

Dilworth gives 11 cases under the head of "Exchange" which grew out of commercial relations between London and other centers of business. The problem is to determine the equivalent in English money of a sum expressed in the money of a foreign place, or the reverse. No explicit directions are given, but evidently the problems are to be solved by the rule of three, or a contraction of it, when a single multiplication or division is possible.

Although the present decimal system of money in the United States was established in 1786, its universal acceptance by the several States was delayed. Prior to 1786 each of the several States had established its own currency. According to Pike they had adopted a common medium of exchange in groups as follows: New Hampshire, Massachusetts, Rhode Island, Connecticut, and Virginia; New York and North Carolina; New Jersey, Pennsylvania, Delaware, and Maryland; South Carolina and Georgia. In addition, Canada and Nova Scotia had another medium of exchange. Each of these five moneys was a modification of the system of English money in pounds, shillings, and pence.

These additional factors were added to the already complex problem of exchange which we have described. Under the head of "Rules for reducing Federal coin and the currencies of the several United States; also English, Irish, Canada, Nova Scotia, Livres, Tournois, and Spanish milled Dollars, each to the par of all the other," Pike gives 76 rules of this type: "To reduce South Carolina and Georgia currency to New Hampshire, Massachusetts, Rhode Island, Connecticut, and Virginia currency. Rule: Multiply the South Carolina, etc., sum by 9 and divide the product by 7." This topic precedes the rule of three, but under this rule additional problems of exchange are given. Later in the text a table of exchange is given for reference.

Daboll gives a similar treatment of exchange, and Adams, in commenting on the improvements contained in the revised edition of his *Scholar's Arithmetic* in 1815, says in the preface:

> But what more particularly claims attention in this revised edition is the introduction of the rule of exchange, where the pupil is made acquainted with the different currencies of the several States (that of South Carolina and Georgia only excepted), and how to change these currencies from one to another; also, to Federal money, and Federal money to these several currencies.

This shows that the needs of business, which are reflected in Pike's treatment of exchange, were felt keenly enough as late as 1815 to cause a revision of a text which had previously omitted that part of exchange which had to do with the currencies of the several States. But, as will appear in the consideration of the texts of the next period, the need was soon removed by the general use of the Federal money.

Percentage.—Problems of loss and gain, discount, interest, etc., which we now class as applications of percentage, are treated in the texts of this period, but the topic of percentage itself does not appear. Cocker (1667) first mentions per cent under profit and loss. After stating and solving three problems having to do with the absolute loss or gain he gives the problem:

A draper bought 86 kerseys for 129 L. I demand how he must sell them per piece to gain 15 L. in laying out 100 L. at that rate.

The answer is justified by saying:

For as 100 is to 115 L. so is 129 L. to 148 L. 7s., so that, by the proportion above, I have found how much he must receive for the 86 kerseys to gain after the rate of 15 L. per C.

He continues his development with the two following problems:

A grocer bought 4¾ C. of pepper for 15 L. 17s. 4d. and (it proving to be demnified) he is willing to lose 12 L. 10s. per cent. I demand how he must sell it per pound.

A plummer sold 10 fodder of lead (the fodder containing 19½ C.) for 204 L. 15s. and gained after the rate of 12 L. 10s. per 100 L. I demand how much it cost him per C.

In these problems and the accompanying explanations Cocker is expressing the loss or gain as being at the rate of so many pounds on each 100 pounds invested. The treatment of loss and gain in the other texts of this period is not quite as explicit, but it is in accord with the spirit of this. Per cent is written "per cent.," which shows that "cent" was clearly understood as an abbreviation for centum. Occasionally "per cent" was contracted to "per C."

Pike introduces profit and loss by saying that it

is an excellent rule by which merchants and traders discover their profit and loss *per cent* or by the gross * * * It also instructs them to raise or fall the price of their goods so as to gain or lose so much *per cent*, etc.

This indicates the recognized function of the topic.

Pike gives this problem under the head of the rule of three: "If 100 L. gain 6 L. in a year, what will 475 L. gain in that time?" But this seems to have been unusual. Dilworth defines "the rate per cent" as "a certain Sum agreed on between the Lender and the Borrower, to be paid for every 100 Pound, for the Use of the Principal, which, according to the Laws of England, ought not to be above 5 L. for the Use of 100 L. for 1 year, and 10 L. for the Use of 100 L. for 2 years; and so on for any Sum of Mony, in Proportion to the time proposed." The rule is, "Multiply the principal by the rate per cent and divide the product by 100, the quotient is the interest required." Pike, Daboll, and Adams give similar definitions and the same rule. For days and months aliquot parts of a year were to be taken. For 6 per cent a special rule was given.

Dilworth treats briefly compound interest and rebate or discount (true discount) in Part I. Later in Part III these topics, together with other applications of percentage, are taken up with decimals. Under the head of "Simple Interest" "the ratio of the rate per cent" is defined as "only the simple interest of 1 L. for one year at any proposed rate of interest per cent." It is to be found by the application of the rule of three thus:

£ £ 1
100 6 . 1 0.06

A table of ratios and the four cases of interest are given, the rule being stated only in term of a formula. No problem is solved out, but presumably decimal fractions are to be employed in solving the problems. Compound interest is presented in the same manner. Annuities and pensions in arrears, present worth of annuities, annuities and leases, and rebate or discount are considered for both simple and compound interest. With only a very few exceptions all possible cases are given. Besides these the topics of purchasing freehold or real estates and purchasing freehold estates in reversion are treated in their several cases. In all cases the rule is stated in terms of a formula.

Pike adds commission, brokerage, partial payments, buying and selling stocks, and policies of insurance as applications or phases of interest. These topics are treated very simply with the exception of policies of insurance, which is given in eight cases. Four of these cases have to do with problems arising in marine insurance.

Both Dilworth and Pike give equation of payments by the common way and by the true way. By the common way the equated time of payment was found by multiplying "each payment by the time at which it" was due and then dividing "the sum of the products by the sum of the payments." The rule for the true way is complicated, but it is based on the recognition that to be absolutely fair interest upon the amounts whose payment is delayed should be equal to the (true) discount upon the amounts which are paid before they are due. Daboll and Adams do not mention equation of payments by the true way. Adams gives a Massachusetts rule for partial payments, and Daboll adds the Connecticut rule.

In the treatment of these several topics which we now associate under the head of the applications of percentage, decimal fractions are used only as a second method. Six per cent always stood for at the rate of 6 L. on 100 L., $6 on $100, 6 cents on 100 cents, etc. To get from 6 per cent to .06 the rule of three was required, and then .06 was called the ratio. In the general organization of the texts after

Dilworth, decimal fractions were placed early enough so that they might have been used directly in the solution of problems which involved "per cent," but in general they were not. The method of solution was accomplished by an application of the rule of three, or directions were given to divide the product by 100 or to cut off two places.

Percentage with its several cases is not contained in the texts. The range of application is as great as we have to-day, but they were handled without the technique of percentage.

Tare and trett had to do with rules for making allowances in the weight of merchandise. Tare was an allowance for the container (box, barrel, bag, etc.). Trett was an allowance of 4 pounds out of each 104 pounds for "waste and dust in some sort of goods." Cloff was an additional allowance of 2 pounds on every 3 hundredweight.

Progressions, arithmetical and geometrical (sometimes called proportion) are treated exhaustively by Pike and partially by Dilworth and Daboll and not at all by Adams. In geometrical progressions the problems are mostly concerning a crafty person who makes an apparently foolish bargain. It involves a geometrical progression, however, and turns out to be most profitable. The merchant who sold 39 yards of fine velvet trimmed with gold at 2 pins for the first yard, 6 pins for the second, 18 pins for the third, etc., is typical.

Permutations was frequently given as a topic. The problems are about such questions as the number of changes which can be rung on a chime of bells, or how many different positions a party can assume at a dinner table. Pike asks how many variations can be made of the alphabet. In Pike's text the topic of combinations is added, and the whole topic elaborated into seven cases. However, the topic is usually very briefly treated.

Evolution.—Dilworth disposes of square root by saying to prepare the given square for extraction "by pointing off every two figures." He gives no further instructions. Cube root he explains in some detail, employing the relation $(a+e)^3 = a^3 + 3a^2e + 3ae^2 + e^3$, and rules are given for finding all roots up to and including the twelfth root. Pike gives an additional method for cube root and a general rule for "extracting the roots of all powers," but does not explicity go beyond the fifth root. He gives also a general method by approximation. Adams considers only square and cube roots. For these he gives elaborate demonstrations which are illustrated by cuts.

Longitude and time.—This had not yet become a topic in the texts of this period. Pike approaches the topic in four problems under the rule of three and in two problems under duodecimals.

Mensuration.—The space given to mensuration varied. Dilworth does not mention it. Pike makes it quite a feature. He introduces practically all of the rules of geometry. (See table of contents in Appendix.) In the other texts it is usually mentioned. The mensurational problems are from commerce rather than from the trades.

Duodecimals.—Part V of Dilworth's text is devoted to duodecimals or cross multiplication. In the preface he states that the topic was not contained in the original text, but added in a revision. Duodecimals are defined as "fractions of a foot, or of an inch, or any part of an inch having 12 for their denominations." Feet, inches, seconds, thirds, and fourths are used. It was the purpose to use a scale of 12 in calculations rather than the decimal scale upon which our number system is based. Other texts of the period give the topic; and the system seems to have been used in practical calculations. Adams speaks of it as a rule which is "particularly useful to workmen and artificers in casting up the contents of their work."

Pleasant and diverting questions.—Arithmetical puzzles are occasionally found mixed in with practical problems. In addition, some authors give a list of puzzles under the above or a similar title. This is the case in the texts by Dilworth and Adams and in their lists we recognize some familiar friends from which we select the following:

Says Jack to his Brother Harry, I can place four threes in such a manner that they shall make just 34, can you do so too?

As I was going to St. Ives,
I met seven wives,
Every wife had seven sacks,
Every sack had seven cats,
Every cat had seven kits,
Kits, cats, sacks and wives,
How many were going to St. Ives?

Three jealous husbands with their wives, being ready to pass by night over a river, do find at the waterside a boat which can carry but two persons at once, and for want of a waterman they are necessitated to row themselves over the river at several times: The question is, how shall these persons pass 2 by 2, so that none of the three wives may be found in the company of one or two men, unless her husband be present?

Proofs.—Dilworth gives what he calls a "proof" for many of the rules of his text. But these "proofs" are rather checks upon the operations than a proof of the rule. A very common form of proof was to reverse the order, i. e., take the answer obtained and work back to the conditions of the problems. In the case of addition and multiplication, it meant to change the order of performing the operation. "Casting out the nines" was used as a method of proving multiplication and addition, but Pike says, following his exposition of this method of proof:

However, the inconveniency attends this method, that, although the work will always prove right, when it is so; it will not always be right when it proves so; I have therefore given this demonstration more for the sake of the curious, than for any real advantage.

Types of problems.—A very small per cent of the total number of problems call for the arbitrary manipulation of abstract numbers, such as, "Multiply 4786 by 753," or, "What is 14 per cent of 8392?" This is due in part to the relatively small space given to the fundamental operations and to the absence of provision for drill upon special rules, such as percentage. There are a number of problems of this type: "Reduce 16 miles to barleycorns." Such problems are essentially abstract, even though they have to do with concrete quantities. They lack the setting of a practical situation which calls for the operation. Instead, the operation is dogmatically demanded. Furthermore, the problems of this type were not always constructed so as to conform to the demands which are made by actual practical situation. For example, it is difficult to imagine the practical situation which would demand the above process.

A somewhat different type of problem, but being practical only in a slightly greater degree, is the following:

From the Creation to the departure of the Israelites from Egypt was 2,513 years; to the siege of Troy, 307 years more; to the building of Solomon's temple, 180 years; to the building of Rome, 251 years; to the expulsion of the kings from Rome, 244 years; to the destruction of Carthage, 363 years; to the death of Julius Cæsar, 102 years; to the Christian era, 44 years; required the time from the Creation to the Christian era?

While such a problem might arise, it is unusual, and the way in which it is stated leaves the pupil to construct the situation which would give rise to such a problem.

A large number of problems, in many texts a majority, were practical in the sense that the statement of the problem included a description of the practical situation which demanded the calculation. But because of the excessive classification of problems under particular rules, the pupil did not need to use his understanding of the practical situation to determine the operations which were required.

The method of presenting a topic.—The manner in which the authors introduced the pupil to arithmetic is typical of the spirit of the texts. In Daboll's *Schoolmaster's Assistant*, which was probably more extensively used in the United States after 1800 than any other arithmetic before Colburn's, the pupil was introduced to the subject as follows:

Arithmetic is the art of computing by numbers, and has five principal rules for its operation, viz, numeration, addition, subtraction, multiplication, and division.

Numeration is the art of numbering. It teaches to express the value of any proposed number by the following characters or figures:

1, 2, 3, 4, 5, 6, 7, 8, 9, 0—or cypher.

Besides the simple value of figures, each has a local value, which depends upon the place it stands in, viz, any figure in the place of units represents only its simplest value, or so many ones, but in the second place, or place of tens, it becomes so many tens, or ten times its simple value.

In the *Scholar's Arithmetic*, by Daniel Adams, the subject was begun as follows:

Arithmetic is the art or science which treats of numbers.

It is of two kinds, theoretical and practical.

The theory of arithmetic explains the nature and quality of numbers, and demonstrates the reason of practical operations. Considered in this sense, arithmetic is a science.

Practical arithmetic shows the method of working by numbers so as to be most useful and expeditious for business. In this sense arithmetic is an art.

There are six pages of definitions of this sort, and an explanation of the system of notation, before any problems are given. Addition is begun with the definition, followed by the rule for addition and for proving the work. The first example is: "What will be the amount of 3612 dolls. 3043 dolls. 651 dolls. and of 3 dollars when added together?" There is nearly a page of explanation. This is followed by nine abstract examples which complete the topic of addition except for a "Supplement to Addition," which was added in the revised edition.

In treating a topic four elements were recognized—definitions, rule, explanation and problems. If the topic permitted being subdivided into cases, this was done. The presentation of the single rule of three direct in Pike's text is perhaps typical.

The Rule of Three Direct teacheth, by having three numbers given, to find a fourth that shall have the same proportion to the third as the second hath to the first.

If more require more, or less require less, the question belongs to the Rule of Three Direct.

But if more require less, or less require more, it belongs to the Rule of Three Inverse.

Rule. 1. State the question by making that number which asks the question the third term, or putting it in the third place; that which is of the same name or quality as the demand, the first term; and that which is of the same name or quality with the answer required, the second term.

2. Multiply the second and third numbers together, divide the product by the first, and the quotient will be the answer to the question, which (as also the remainder) will be in the same denomination you left the second term in, and which may be brought into any other denomination required.

Two or more statings are sometimes necessary, which may always be known from the nature of the question.

The method of proof is by inverting the question.

But, that I may make the method of working this excellent rule as intelligible as possible to the learner, I shall divide it into the several cases following:

1. The fourth number is always found in the same name in which the second is given, or reduced to; which, if it be not the highest denomination of its kind, reduce to the highest, when it can be done.

2. When the second number is of divers denominations, bring it to the lowest mentioned, and the fourth will be found in the same name to which the second is reduced, which reduce back to the highest possible.

3. If the first and third be of different names, or one or both of divers denominations, reduce them both to the lowest denomination mentioned in either.

4. When the product of the second and third is divided by the first; if there be a remainder after the division, and the quotient be not the least denomination of its

kind; then multiply the remainder by that number, which one of the same denomination with the quotient contains of the next less, and divide this product again by the first number; and thus proceed 'till the least denomination be found, or 'till nothing remain.

5. If the first number be greater than the product of the second and third, then bring the second to a lower denomination.

6. When any number of barrels, bales, or other packages or pieces are given, each containing an equal quantity, let the content of one be reduced to the lowest name, and then multiply by the given number of packages or pieces.

7. If the given barrels, bales, pieces, etc., be of unequal contents (as it most generally happens), put the separate content of each properly under one another, then add them together, and you will have the whole quantity.

The organization of the texts.—The plan of organization of the texts of this period was topical, and there was considerable agreement in the general arrangement of the topics. Numeration was uniformly the first topic. Sometimes notation was also included under this head, as in Pike's arithmetic, or it was given as a separate topic as in Adams's *Scholar's Arithmetic*. This was followed by the four fundamental operations for integers, and these in turn by denominate numbers, including reduction, compound addition, etc. Federal money came to have a place early in the text. Fractions, both vulgar and decimal, came late in the first texts (see tables of contents in Appendix). In later texts (Pike, Daboll, and Adams) they immediately followed compound numbers. The rule of three in its variety of forms was placed after fractions, and in turn it was followed by practice, tare and trett, exchange, interest, brokerage, etc.

Adams has a rather unique general plan of organization. After disposing of numeration and notation he divides the remaining content into four sections as follows:

Section I. Fundamental Rules of Arithmetic.

Section II. Rules essentially necessary for every person to fit and qualify them for the transaction of business.

Section III. Rules occasionally useful to men in particular employments of life.

Section IV. Forms of notes, &c.

Specifying the rules of Section II, he says:

These are ten: Reduction, fractions,[1] Federal money, exchange, interest, compound multiplication, compound division, single rule of three, double rule of three, and practice.

A thorough knowledge of these rules is sufficient for every ordinary occurrence in life.

A summary.—The most prominent feature of the arithmetic texts of this period was the large number of rules which were to be directly applied to problems of trade and commerce. The texts were essentially commercial arithmetics. The solutions of the exercises were to be written as opposed to oral and the subject was frequently called "cyphering" for this reason.

[1] Fractions are taken up here no further than is necessary to show their signification, and to illustrate the principles of Federal money.

Primary and mental arithmetic.—The texts which we have described were not intended for young children. In fact, during this period arithmetic was seldom taught to children before the age of 8 to 10. The content was limited to written calculations and mental arithmetic did not exist. Between 1800 and 1821 there were a few attempts to make arithmetic "easy" for children.[1] The general plan of these texts was the same as that of the ones we have described. Some of the topics were omitted, and the treatment of the others was less elaborate. They possessed none of the essential features of the primary texts of the next period. There was one attempt at a mental arithmetic.[2] The text was mental only in the sense that rules and facts were to be memorized and short cuts emphasized so that the pupil could make the calculations mentally. There were no analyses. It was not a mental arithmetic of the type which became prominent a few years later.

The appearance of these texts is significant in that it indicates a tendency which culminated in Colburn's *First Lessons*, which is described in Chapters IV and V.

Texts not exclusively devoted to arithmetic.—In addition to texts of the type which we have described, there were a number of books which were encyclopedic in the range of topics treated. They were usually published under such titles as, "Instructor," "Companion," "Assistant," etc. The following complete title of a book of this type gives a good idea of the range of topics. Out of a total of 384 pages, 94 are devoted to arithmetic, not including bookkeeping.

The Instructor; or, American Young Man's Best Companion. Containing, S[illegible]ing, Reading, Writing and Arithmetic, in an easier way than any yet published; and how to qualify any person for business, without the help of a Master. Instructions to write Variety of Hands, with Copies, both in Prose and Verse. How to write Letters on Business or Friendship. Forms of Deeds, Bonds, Bills of Sale, Powers of Attorney, Indentures, Receipts, Wills, Leases, Releases, etc.

Also, Merchants' Accounts, and a short and easy Method of Shop and Bookkeeping; with a Description of the Product, Counties and Market Towns in England and Wales, etc.

Together with the Method of Measuring Carpenters', Joiners', Sawyers', Bricklayers', Plasterers', Plumbers', Masons', Glaziers', and Painters' Work. How to undertake each Work. With the Description of Gunter's Line and Coggershall's Sliding-Rule.

Likewise, the Practical Gauger, Made Easy; the art of Dialling; How to erect and fix Dials; with Instructions for Dying, Colouring, and making Colours; and some General Observations for Gardening every Month in the year.

To which is added, The Family's Best Companion: With Instructions for marking on Linen; How to Pickle and Preserve; to make divers Sorts of Wine; and many excellent Plasters, and Medicines, necessary in all Families; and a Compendium of

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[1] Examples of such texts are: Arithmetic Made Easy to Children, by Emmor Kimber, second edition, 1805; An Arithmetical Primer for Young Masters and Misses, by Samuel Temple, 1809.

[2] John White: A Practical System of Mental Arithmetic, or a New Method of Making Calculations by the Action of the Mind without Pen, Ink, Pencil, or Paper. 1818.

the Sciences of Geography and Astronomy containing a brief Description of the different Parts of the Earth, and a Survey of the Celestial Bodies.

Also Several Very Useful Tables.

By George Fisher, Accomptant, 30th Ed. Revised, Corrected, Enlarged, Improved. Printed at Worcester, Mass., 1785, by I. Thomas.

The contribution of the period.—The sixteenth century has been called the great constructive period in the development of arithmetic. A comparison of the texts used in the United States before 1821 with the description which L. L. Jackson gives in *The Educational Significance of Sixteenth Century Arithmetic* shows no important advance over the arithmetic of the sixteenth century except the introduction of decimal fractions. But only partial use was made of decimal fractions, even after the introduction of Federal money, until after the close of this period. "Rules reducing Federal coin, etc.," represent the only important addition to the contents of Dilworth's *Schoolmaster's Assistant* and earlier texts by English authors.

The contributions of the ciphering book period to the development of arithmetic as a school subject were (1) the arousing of interest in arithmetic suited to the needs of the United States, and (2) the cultivation of a sense of discrimination in the selection and organization of the subject matter of arithmetic. The first is shown by the number of texts by American authors which were published between 1788 and 1821. The second is shown in a general way by all the texts. In particular it is shown by the contents and form of organization of Adams's *Scholar's Arithmetic* and in Daboll's attitude toward fractions. These two contributions are significant not so much for what was accomplished during this period as for the fruit they bore in the next.

The content of the instruction.—When the arithmetic actually taught is considered, it must be noted that in the latter portion of this period something of arithmetic was taught in four types of schools; Dame schools, public schools, private schools, and academies and colleges. These will be considered in order.

Dame schools.—Dame schools, as the name signifies, were kept by women, usually maiden ladies, who were willing for a small competence to care for very young children, thereby relieving their mothers of that burden. This was the main function of the dame schools, but in addition the children were usually taught their letters and occasionally to read. Often no more was possible because of the very limited ability of the one who gave the instruction. Occasionally the children were taught to count and to chant the addition and multiplication tables. With the exception of the most pretentious of these schools, the instruction in arithmetic did not extend beyond this.

Public schools.—The arithmetic studied by pupils in these schools seems to have been often confined to the four fundamental operations with integers and to these operations for denominate numbers. The following is typical of a number of statements relative to the latter part of this period:

Arithmetic was taught from Dilworth, a book making no allusion to a decimal currency, and having little or no adaptation to the ordinary requirements of business. If we reached the "Rule of Three," we were quite gratified with our attainments. Most of us came short of it.[1]

It should be noted that in Dilworth's Schoolmaster's Assistant the rule of three began on page 129, immediately following reduction, which was preceded by only notation and the four operations for integers. Wickersham says:

Before 1800, he was considered a remarkable scholar who in a country school had ciphered beyond the rule of three, and few schoolmasters made pretension to a knowledge of arithmetic more extensive.[2]

Warren Burton in *The District School as It Was* tells us that:

My third season I ciphered to the very last sum in the rule of three. This was deemed quite an achievement for a lad only 14 years old, according to the ideas prevailing at that period. Indeed, whoever ciphered through the above-mentioned rule was supposed to have arithmetic enough for the common purposes of life. If one proceeded a few rules beyond this, he was considered quite smart. But if he went clear through—miscellaneous questions and all—he was thought to have an extraordinary taste and genius for figures. Now and then, a youth, after having been through Adams, entered upon old Pike, the arithmetical sage who "set the sums" for the preceding generation. Such were called great "arithmeticians."[3]

In New York City in 1815 we are told that, of the children studying arithmetic, 208 were in addition and subtraction, 110 in multiplication and division, 15 in compound numbers, and only 10 in reduction and the rule of three.[4]

Private schools.—In general these were schools for giving instruction in special subjects, such as arithmetic and writing. Hence the instruction in arithmetic was more pretentious than that in the public schools. Advertisements of schoolmasters of about the middle of the eighteenth century have been preserved and are indicative of the scope of the instruction in these schools. These are some of the less pretentious:

From the Newport Mercury of May 22, 1759: "John Sims, schoolmaster in the town school, teacheth reading and writing, arithmetic, both vulgar and decimal, geometry, trigonometry, and navigation, with several other branches of Mathematics."

Another notice from the same paper, under date of December 19, 1758, states:

"Sarah Osborne, schoolmistress in Newport, proposes to keep a boarding school. Any person desirous of sending children may be accommodated and have them

[1] Barnard: Amer. Jour. of Educ., 16: 738.

[2] J. P. Wickersham; History of Education in Pennsylvania, p. 192.

[3] Pp. 110-113.

[4] Barnard; Amer. Jour. of Educ., 19: 509.

instructed in reading, writing, plain work, embroidering, tentstitch, samplers, etc., on reasonable terms."[1]

A few schoolmasters, however, were more pretentious in their announcements.

In 1746 the master of the Anne Arundel County School was John Wilmot, who concisely and expeditiously taught "reading, writing in the most usual hands, grammar, arithmetic, vulgar, decimal, instrumental, algebraical, merchant's accounts, with the Italian method of bookkeeping, geometry, trigonometry, plain and spheric, with their applications, surveying, navigation, astronomy, dialing, likewise the use of the globes, and sundry other parts of mathematics."[2]

Peter Robinson, at Upper Marlboro, near which place youth may be boarded, taught "reading, writing in all hands, arithmetic in whole numbers and fractions, vulgar and decimal, also artificial arithmetic, both logarithmical and logistical, with instrumental either by inspection, rabdologia or proportional scales, geometry, both superficial and solid, with mensurations of all kinds, either in longimetria, planometria, or stereometry, as surveying, fortification, gunnery, gauging, etc ; trigonometry, both plain and spherical, with navigation either in plain, mercator, or circular sailing, also dialing, all sorts and ways, either arithmetically, geometrically, projective, reflective, concave, or convex; cosmography, celestial or astronomical, and terrestrial or geographical; astronomy, practical and theoretical; grammar, merchant's accounts, or the art of bookkeeping after the Italian manner; algebra, Euclid's elements, etc.; likewise the description and use of sea charts, maps, quadrants, fore staffs, nocturnal, protractor, scales, Coggershall's rule, sector, gauging rod, universal ring dial, globes, and other mathematical instruments".[2]

A ciphering book executed by Miss Catharine G. Willard while she was attending the Ladies Academy, Boston, 1809, contains the following topics: Numeration, addition, subtraction, multiplication, division, compound numbers (pence table, lawful money, Federal money, Troy weight, avoirdupois weight, apothecaries weight, cloth measure, long measure, square measure, cubic measure, wine or beer measure, ale measure, time measure), addition, subtraction, multiplication, and division of compound numbers, reduction by division, reduction by multiplication, single rule of three direct, single rule of three inverse, double rule of three, single fellowship, double fellowship, simple interest, compound interest, exchange, practice (20 cases), bills of parcels, discount, barter, tare and trett, loss and gain, equation of payments, commission, brokerage, buying and selling stocks, and policies of insurance.

Few ciphering books are accessible in libraries, but such as it has been possible to examine indicate that the range of topics was generally less than in this one, although one was examined which included even a greater number of topics.

Academies and colleges.—In the academies and colleges such a text as Pike's was used, and something of the science of arithmetic as well as the art of ciphering was attempted. The students were more mature and they were able to advance beyond the pupils receiving instruction in the public schools or even in most of the private schools. In 1804, 16 academies reported to the regents of New York 963 stu-

[1] W. H. Tolman: Hist. of Higher Educ. in Rhode Island, p. 20.
[2] B. C. Steiner: Hist. of Educ. in Maryland, p. 33.

dents. Of this number, 429 were studying grammar and arithmetic. In 1801 a two-volume treatise on mathematics by Samuel Webber, Hollis professor of mathematics and natural philosophy, of Harvard, was published. Of the first volume 245 pages are devoted to arithmetic. In an advertisement in the second edition it is stated that "The design in making this compilation is to collect suitable exercises to be performed by the classes at the private lecture on mathematics given in the university (Harvard)." There is a close correspondence between the topics on arithmetic and those found in the first 345 pages of Pike's arithmetic, but the text is even more advanced than Pike's. It is noticeably more abstract and philosophical. Numerous footnotes contain explanatory material, often concerning the reason for a rule.

Chapter III.

THE CIPHERING BOOK METHOD OF TEACHING ARITHMETIC.

The ciphering book.—The most conspicuous feature of the manner of teaching arithmetic during this period is the absence of texts in the hands of the pupils. It was the unusual exception for a pupil to possess any sort of an arithmetic text until toward the close of this period. Even when a pupil possessed a text the usual plan of instruction does not appear to have been altered. In fact, as we shall show, many of the early texts were specific attempts to facilitate and not to change the plan of teaching.

When a pupil was thought old enough and desired to learn to "cypher," he provided himself with a blankbook which was made of a quire of paper folded and sewed together. This was his "cyphering book." The children of the more well-to-do parents often had "bought" ciphering books. The paper of these was of good quality, and the books were bound with board or leather covers. In appearance they resembled account books of to-day, and doubtless they excited the envy of those who were compelled to content themselves with the home-made product.

Having been equipped with a ciphering book, the pupil announced his desire to the master, provided instruction in arithmetic was given in the school. If not, he must go elsewhere for this part of his education. In either case the procedure was the same. The master usually possessed a ciphering book which he had made when he learned to cipher. He set the pupil a "sum," often the first one in his own ciphering book, and told him the rule for its solution. The pupil was then to work out his own salvation. One pupil described his experience as follows:

> At length, in 1790 or 1791, it was thought that I was old enough to learn to "cypher," and accordingly was permitted to go to school more constantly. I told the master I wanted to learn to cypher. He set me a "sum" in simple addition—*five columns* of figures and *six figures* in each column. All the instruction he gave me was, Add the figures in the first column, carry one for every ten, and set the over-plus down under the column. I supposed he meant by the *first* column the left-hand column, but what he meant by carrying one for every ten was as much a mystery as Samson's riddle was to the Philistines. I worried my brains for an hour or two, and showed the master the figures I had made. You may judge what the amount was when the columns were added from left to right. The master frowned and repeated his former instruction, Add up the column *on the right*, carry one for every ten, and set down the remainder. Two or three afternoons (I did not go to school in the morning) were

spent in this way, when I begged to be excused from learning to cypher, and the old gentleman with whom I lived thought it was time wasted; * * * The next winter there was a teacher more communicative and better fitted for his place, and under him some progress was made in arithmetic, and I made a tolerable acquisition in the first four rules, according to Dilworth's *Schoolmaster's Assistant*, of which the teacher and one of the eldest boys had each a copy. The two following winters, 1794 and 1795, I mastered all the rules and examples in the first part of Dilworth; that is, through the various chapters of rule of three, practice, fellowship, interest, etc., to geometrical progression and permutation.[1]

The example or problem was worked on a scrap of paper, often coarse wrapping paper. Later slates were used. After the pupil had finished the "sum" to his own satisfaction, he carried his work up to the master's desk for approval. The approval frequently consisted in the master comparing the pupil's work with his own in his ciphering book. This solution had received his master's stamp of approval in this same way and probably could boast of several generations of unbroken descent. If the pupil's work was identical with the master's, it was approved and was ordered copied, together with the "rule," in the ciphering book, to be preserved. If there was not identity, the pupil was frequently commanded to "do it all over again," even though his work was correct. One pupil describes his experience thus:

Printed arithmetics were not used in the Boston schools till after the writer left them,[2] and the custom was for the master to write a problem or two in the manuscript of the pupil every other day. No boy was allowed to cypher till he was 11 years old, and writing and cyphering were never performed on the same day. Master Tileston had been taught by Master Proctor, and all the sums he set for his pupils were copied exactly from his old manuscript. Any boy could copy the work from the manuscript of any further advanced then himself, and the writer never heard any explanation of any principle of arithmetic while he was at school. Indeed, the pupils believed that the master could not do the sums he set for them, and a story is told of the good old gentleman, which may not be true, but which is so characteristic as to afford a very just idea of the course of instruction, as well as of the simplicity of the superannuated pedagogue.

It is said that a boy, who had done the sum set for him by Master Tileston, carried it up, as usual, for examination. The old gentleman, as usual, took out his manuscript, compared the slate with it, and pronounced it wrong. The boy went to his seat and reviewed his work, but finding no error in it returned to the desk and asked Mr. Tileston to be good enough to examine the work, for he could find no error in it. This was too much to require of him. He growled, as his habit when displeased, but he compared the sum again, and at last, with a triumphant smile, exclaimed: "See here, you nurly (gnarly) wretch, you have got it 'If four tons of hay cost so much, what will seven tons cost?' when it should be, 'If four tons of *English* hay cost so and so.' Now go and do it all over again."[3]

The origin of this plan of teaching probably was due to the scarcity of texts. But the continuance of a method of instruction which did not make use of a text can not be due to texts not being available

[1] Joseph T. Buckingham, in Barnard's Amer. Jour. of Educ., 13: 130.

[2] John Tileston taught in the North Writing School of Boston from 1762 to 1819. Wm. B. Fowle lived from 1795 to 1865. This would place the date of this about 1810.

[3] Wm. B. Fowle, in Barnard's Amer. Jour. of Educ., 5: 338.

during the eighteenth century. Several texts by English authors were known in the Colonies in the early part of the eighteenth century. (See p. 18.) That these texts did not become generally used in the schools and the method of instruction correspondingly changed seems to have been due mainly to a belief in the efficacy of the method of teaching arithmetic without a text. To be sure, books were more expensive then than now, if the purchasing value of money be taken into account. But this condition does not seem to have prevented the printing and distribution of religious writings and the almost universal use of the Psalter and catechism in the schools. And certainly in the study of Latin texts must have been used. There were a number of printing presses in the Colonies, but they were engaged largely in turning out controversial pamphlets of a religious nature.

When pupils came to be provided with texts, the procedure was essentially the same until after the close of this period. Wickersham gives this description:

> With a book of his own, the pupil solved the problems contained in it in their proper order, working hard or taking it easy as it pleased him, showed the solutions to the master, and if found correct generally copied them in a blank book provided for the purpose. The matter copied embraced about the whole contents of the arithmetic, including headings, definitions, rules, and examples. Some of these old manuscript "Cyphering Books," the best one may suppose, having come down through several generations, are still preserved among old family records, bearing testimony to the fair writing and the careful copying, if not to the arithmetical knowledge, of those who prepared them. When a pupil was unable to solve a problem, he had recourse to the master, who solved it for him. It sometimes happened that a dozen or twenty pupils stood at one time in a crowd around the master's desk, waiting with slates and problems to be solved. There were no classes in arithmetic, no explanations of processes either by master or pupil, no demonstration of principles either asked for or given; the problems were solved, the answers obtained, the solutions copied, and the work was considered complete. That some persons did obtain a good knowledge of arithmetic under such teaching must be admitted, but this result was clearly due rather to native talent or hard personal labor than to wise direction.[1]

Another interesting account of the teaching of arithmetic is given in *The District School as It Was*. The writer was born in 1800 and the account was first published in 1833:

> At the age of 12 I commenced the study of arithmetic, that chiefest of sciences in Yankee estimation. No man is willing that his son should be without skill in figures * * *.
>
> The entering on arithmetic was quite an era in my schoolboy life. This was placing me decidedly among the great boys and within hailing distance of manhood. My feelings were consequently considerably elevated. A new Adams's arithmetic of the latest edition was bought for my use. It was covered by hand with stout sheepskin, in the economical expectation that, after I had done with it, it might help still younger heads to the golden science. A quire of foolscap was made to take the form of a manuscript of the full length of the sheet, with a pasteboard cover, as more suitable to the dignity of such superior dimensions than filmy brown paper.

[1] Hist. of Educ. in Pennsylvania, pp. 205-206.

I had also a brand-new slate, for Ben used father's old one. It was set in a frame wrought by the aforesaid Ben, who prided himself on his knack at tools; considering that he had never served an apprenticeship in their use. There was no lack of timber in the fabrication. Mark Martin said that he could make a better frame with a jack-knife in his left hand and keep his right in his pocket.

My first exercise was transcribing from my arithmetic to my manuscript. At the top of the first page I penned "ARITHMETIC" in capitals an inch high and so broad that this one word reached entirely across the page. At a due distance below I wrote the word "Addition" in large coarse hand, beginning with a lofty A, which seemed like the drawing of a mountain peak, towering above the level wilderness below. Then came "Rule," in a little smaller hand, so that there was a regular gradation from the enormous capitals at the top, down to fine running, no, hobbling hand in which I wrote off the rule.

Now slate and pencil and brain came into use. I met with no difficulty at first; simple addition was as easy as counting my fingers. But there was one thing I could not understand—that carrying of tens. It was absolutely necessary, I perceived, in order to get the right answer; yet it was a mystery which that arithmetical oracle, our schoolmaster, did not see fit to explain. It is possible that it was a mystery to him. Then came subtraction. The borrowing of ten was another unaccountable operation. The reason seemed to me then at the very bottom of the well of science; and there it remained for that winter, for no friendly bucket brought it up to my reach.

Every rule was transcribed to my manuscript, and each sum likewise as it stood proposed in the book, and also the whole process of figures by which the answer was found.

Each rule, moreover, was, or rather was to be, committed to memory, word for word, which to me was the most tedious and difficult job of the whole.[1]

The purpose of texts in the hands of pupils was to lighten the labor of the master as well as to assist the pupil in understanding the subject. Dilworth says in his preface:

After returning to you my most hearty thanks for your kind acceptance of my *New Guide to the English Tongue*, permit me to lay before you the following pages, which are intended as a help toward a more speedy improvement of your scholars in numbers and at the same time to take off that heavy burden of writing out rules and questions which you have so long laboured under.

And later:

And now, after all, it is possible that some, who like best to tread the old beaten Path, and to sweat at their Business when they may do it with Pleasure, may start an Objection against the Use of this well intended Assistant.

Another author writes in more detail and lets us see more of the schoolroom technique and the reason for having printed texts:

Having some time ago drawn up a set of rules and proper questions, with their answers annexed, for the use of my own school and divided them into several books, as well for more ease to myself as the readier improvement of my scholars, I found them by experience of infinite use; for when a master takes upon him that laborious (though necessary) method of writing out the rules and questions in the children's books he must either be toiling and slaving himself, after the toil of the school is over, to get ready the books for the next day, or else he must lose that time which would be much better spent in instructing and opening the minds of his pupils. * * * There was, however, still an inconvenience which hindered them from giving me the satis-

[1] Warren Burton: The District School as It Was, pp. 110-112.

faction I at first expected; i. e., where there are several boys in a class, some one or other must wait till the boy who first has the book finishes the writing out of those rules and questions he wants, which detains the others from making that progress they otherwise might, had they a proper book of rules and examples for each; to remedy which I was prompted to compile one, in order to have it printed, that it might not only be of use to my own school but to such others as would have their scholars make a quick progress.[1]

Daniel Adams says in the preface to his Scholar's Arithmetic:

We have now the testimony of many respectable teachers to believe that this work, where it has been introduced into schools, has proved a very kind assistant toward a more speedy and thorough improvement of scholars in numbers and at the same time has relieved masters of a heavy burden of writing out rules and questions under which they have so long labored to the manifest neglect of other parts of their schools.

The *Pupil's Guide*, by Benjamin Dearborn (1782), is simply a collection of the "most useful rules in arithmetic." The book contains no examples or problems. Its purpose was "to lessen the labor of the master."

A few authors attempted to facilitate this plan of instruction even more than by simply providing a source for problems and rules. Isaac Greenwood, the first American to write an arithmetic which was published, says in his preface:

The Reader will observe, that the author has inserted under all those rules, where it was proper, Examples with Blanks for his practice. This was a principal End to the Undertaking; that such persons as were desirous thereof might have a comprehensive Collection of all the best Rules in the Art of Numbring, with examples wrought by themselves. And that nothing might be wanting to favour this design, the Impression is made upon several of the best Sorts of Paper. This method is entirely new, * * *

Daniel Adams embodies this same feature in his Scholar's Arithmetic in 1801. He says in his preface:

To answer the several intentions of this work it will be necessary that it should be put into the hands of every arithmetician; the blank after each example is designed for the operation by the scholar, which being first wrought upon a slate or waste paper he may afterwards transcribe into his book.

This text apparently attained some popularity, for by 1815 it had gone through nine editions. I have seen a copy printed as late as 1824. I have a copy printed in 1820 which bears the imprint of the hand of some pupil who doubtless labored long over the involved and obscure exercises.

When Dudley Leavitt edited an abridged edition of Pike's System of Arithmetic in 1826, there was also published—

A New Ciphering Book, adapted to Pike's Arithmetic abridged; containing illustrative notes, a variety of useful Mathematical Tables, etc., with blank pages of fine paper, sufficient for writing down all the more interesting operations.[2]

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[1] Francis Walkingame's Tutors' Assistant, preface. This is a text by an English author, but was reprinted in this country.

[2] Amer. Jour. of Educ., 1826, p. 311.

Texts in the hands of the pupils grew in favor, and thus the masters were relieved from the burden of setting sums and dictating rules. The pupil had both in his text. But in doing this another need was created. Many of the instructors possessed practically no arithmetical ability. So little, that the pupils believed that the masters could not do many of the sums, and without their own ciphering books would be helpless. Thus when a new or different problem appeared in the texts, master and pupil alike were perplexed if they could not locate it under a known rule. The following is typical of what one may imagine happened frequently:

A law had just been passed requiring that teachers' examinations should be conducted by three county commissioners instead of the township trustees, as had been the practice before. "I shall not forget," says Hobbie, "my first experience under the new system. The only question asked me at my first examination was, What is the product of 25 cents by 25 cents? We were not as exact then as people are now. We had only Pike's arithmetic, which gave the sums and the rules. These were considered enough at that day. How could I tell the product of 25 cents by 25 cents, when such a problem could not be found in the book? The examiner thought it was 6¼ cents, but was not sure. I thought just as he did, but this looked too small to both of us. We discussed its merits for an hour or more, when he decided that he was sure I was qualified to teach school, and a first-class certificate was given me."[1]

As will be shown a little later, new types of texts began to appear in 1821. To make possible their use, it was necessary to provide a key for the use of the teacher. A key, bound either with the text or separately, became an essential part of a series of arithmetics.

The use of the ciphering book is so conspicuous in the plan of instruction and in the purpose of the texts of this period that it seems appropriate to call the period in the development of arithmetic up to 1821 the "Cyphering Book Period."

Although the ciphering book represents the most conspicuous feature of the teaching of arithmetic during this period, a careful analysis of the method of teaching reveals other factors of fundamental importance.

From the abstract to the concrete.—We have shown that the texts were organized upon this principle. It represents also the order of the instruction. The experience of the boy who was started on a "sum in simple addition—*five columns* of figures, and *six figures* in each column"—seems to be typical.

In his Scholar's Arithmetic Adams presents abstractly the four operations for integers and addition and subtraction for "compound numbers." Following the completion of this section of the text, he says:

The scholar has now surveyed the ground work of arithmetic. It has before been intimated that the only way in which numbers can be affected is by the operations of addition, subtraction, multiplication, and division. These rules have now been

[1] Quoted by F. Cajori, The Teaching and History of Mathematics in the United States, p. 16.

taught him, and the exercises in a supplement to each suggest their use and application to the purposes and concerns of life. Further, the thing needful, and that which distinguishes the arithmetician, is to know how to proceed by application of *these four rules* to the solution of any arithmetical question. To afford the scholar this knowledge is the object of all the succeeding rules.[1]

Not only was the above dictum literally followed, but it manifested itself in the form of proceeding from the general to the particular. It appears that educators of those days really thought this the proper order.

Memoriter method.—As must necessarily be the case when the above order is followed, the memory was emphasized almost exclusively. This was certainly true in this early period. Adams probably only expresses the consensus of opinion when he says: "Directions to the Scholars: Each rule is first to be committed to memory; afterwards, the examples in illustration, and every remark is to be perused with care." [2]

A very vivid description of the method of teaching arithmetic is given by another author:

The boy, advanced perhaps some way in his teens, is sent to a winter school for two or three months to complete his education; for he can not attend in any other season, nor then indeed but quite unsteadily. But as he is almost a man he must go to school to *cypher;* and as he has but a short time for the business he must cypher fast. He goes to school, vulgarly speaking, *raw*, perhaps scarcely able to form an arithmetical figure. His master sets him a sum in addition, and it may be tells him he must carry one for every ten; but why, is a mystery which neither master nor scholar gives himself any trouble about; however, with a deal of pains, he at length gets his sum done, without ever being asked, or knowing how to read the sum total, or any number expressed in the statement.[3]

But it is cyphering, and that is sufficient. If he is taught to commit any of the rules to memory, he learns them like a parrot, without any knowledge of their reason, or application. After this manner he gropes along from rule to rule, till he ends his blind career with the rule of three; and in the end, the only an[illegible]est account he can give of the whole is, that he has been over it. But he has completed his school education, and is well qualified to teach a school himself the next winter after.[4]

The idea of assisting the pupil not a part of the teacher's creed.—In these early days the function of the teacher was to maintain order and hear lessons. The doctrine of interest had not yet been promulgated in this country, nor was it considered necessary to motivate the work of the school, except, perhaps, by punishment in case the lessons were not satisfactorily prepared. In the accounts of the teaching in these early days we find no mention of attempts to assist the pupil to appreciate arithmetic. The study of arithmetic was not compulsory during the greater part of this period. The pupil did not undertake the study until he desired to do so, and pre-

[1] P. 50.

[2] Scholar's Arithmetic, p. 7.

[3] As a striking example of this method of instruction, I have actually known a lad of 18, who, after having in this way, gone over all the first rules of arithmetic at a common school, was utterly unable to read or enumerate any number consisting of four places of figures.

[4] Chauncey Lee: The American Accomptant, preface.

sumably studied it only so long as it pleased him, or possibly his parents. Certain incentives combined to excite interest in the subject. For example, ciphering was a relatively rare accomplishment, particularly until after 1800. Because of the few who could claim the title of "arithmeticker," the title carried with it considerable distinction and honor as well as some practical advantages. Also, the subject was relatively new and considered difficult. These conditions combined to cause rivalry, and one can easily imagine the zeal with which the children to whom such a thing appealed attacked sums which had a reputation of being difficult. Under these conditions the need of motivating the work in arithmetic would not be felt as keenly as under our present conditions. But when interest in the work flagged, as it must have at times, or when a pupil failed to become interested, we have no evidence that it was considered the teacher's function to stimulate interest except by one means, i. e., punishment. In some cases the pupil was allowed to drop the subject.

The master set or dictated sums and rules and examined the pupil's work, or where the pupil possessed a text he had only to examine the work. These were the essential features of the instruction. In the case of some teachers they represented the total of instruction; other teachers were "more communicative." Just what this quality was we are not told, but we may draw some conclusions from the texts of the period which were presumably written by some of the "more communicative" teachers. In the text the assistance given to the pupil is limited to an explanation, or demonstration, of an example which has been selected to illustrate a rule. The explanation is little more than an elaboration of the rule for this special case. The following explanation of the first example in division in Adams's Scholar's Arithmetic is typical. The example is, Divide 127 by 5.

> Proceed in this operation thus: It being evident that the divisor (5) can not be contained in the first figure (1) of the dividend, therefore assume the first two figures (12) and inquire how often 5 is contained in 12; finding it to be 2 times, place 2 in the quotient, and multiply the divisor by it, saying 2 times 5 is 10, and place the sum (10) directly under 12 in the dividend. Subtract 10 from 12 and to the remainder (2) bring the next figure (7) at the right hand, making the remainder 27. Again, inquire how many times 5 in 27; 5 times; place 5 in the quotient, multiply the divisor (5) by the quotient figure (5), saying 5 times 5 is 25, place the sum (25) under 27, subtract, and the work is done. Hence it appears that 127 contains 5, 25 times with a remainder of 2, which was left after the last subtraction.[1]

Such assistance is "telling" with no attempt at development. The attitude is: The rule is difficult because it is not finely enough divided. Hence we will state it for a particular case in more detail. No attempt to develop the topic or to teach the pupil to think is indicated.

[1] P. 27.

Deductive instruction.—It has just been shown that there was practically no attempt to instruct pupils, but in so far as there was any plan for guiding the pupil in his learning, it was deductive, i. e., from rule to problem. The textbooks were arranged on this plan, and the evidence indicates that school practice followed this order exclusively.

No drill.—As has been seen, the texts made practically no provision for drill, and ciphering books of the period indicate that no drill was given. Speed contests and rapid drills were wanting. A partial reason is that blackboards and slates were unknown until near the close of this period. Paper was expensive, and the manufacture and repair of quill pens took much time. There appears to have been no drill, even upon the multiplication tables, except in the dame schools. At this stage of his education the pupil could not write, and hence his work must be oral, and drill upon the number facts was a feature of it.

No oral arithmetic.—The work was all written except the very elementary instruction in number given in the dame schools. The beginning of oral arithmetic, or as it is more generally called, mental arithmetic, belongs to the next period.

Individual instruction.—The very nature of the ciphering book method made impracticable class or group instruction. Though little instruction was given, the pupils had individual contact with the master. The practice of having examples explained to the class by either the teacher or a pupil comes in a later period.

An objective result.—The accumulating collection of examples and their solutions furnished tangible evidence of the pupil's progress. By both master and pupil this objective result was consciously striven for. Incidentally, neatness both in the making of characters and in the arrangement of the work was insisted upon and usually secured. Ciphering books which are still preserved are models of neatness and skill in writing.

PART II.—BEGINNING OF THE PESTALOZZIAN MOVEMENT IN AMERICA AS APPLIED TO ARITHMETIC, OR THE WORK OF WARREN COLBURN.

Chapter IV.

WARREN COLBURN AND HIS RELATION TO PESTALOZZI.

The beginning of a new epoch in arithmetic and arithmetic teaching in the United States was marked by the appearance of Warren Colburn's *First Lessons in Arithmetic on the Plan of Pestalozzi*, in 1821. Arithmetic was given a place of increased importance as a school subject; the content of the texts was changed almost abruptly; the aim of arithmetical instruction was modified to include mental training as an important factor; and much of the instruction in arithmetic became oral. Warren Colburn exerted a greater influence upon this development of arithmetic in the United States than any other person. For this reason it is entirely fitting that we give here a brief account of his life.[1]

Warren Colburn's early life.—Warren Colburn was born March 1, 1793, in the part of Dedham (Mass.) called Pond Plain. In 1794 or 1795 the family moved into Clapboardtrees parish, later to High Rock, and in 1800 or 1801 to Milford. Richard Colburn, his father, was a farmer, and the early life of the boy was spent on the farm. Presumably he participated in the usual activities of farm life. At the age of 4, Warren attended a summer district school. At Milford, he began to attend the winter district school. From Milford the family moved, about the year 1806, to Uxbridge, where he continued to attend the winter terms of the common school.

It was at this last place that his aptitude and expertness in arithmetic began to attract attention. His father encouraged this aptitude by taking into the family Mr. Gideon Alby, an old schoolmaster, who was good at figures. Mr. Alby instructed the boy in "cyphering" during the long winter evenings.

About this time Warren Colburn seems to have developed either a distaste for the farm, or an aptitude for machinery and certain lines of manufacturing. Apparently on his account, the family left the farm about 1810 and moved to Pawtucket, R. I., so that he might have the opportunity to learn something of machinery and manufac-

[1] The source of most of the facts of Warren Colburn's life is an account in Barnard's *American Journal of Education*, 2: 294.

turing. During the next five years he worked in factories, and it was not until the summer of 1815 that he began to prepare himself for college. Just why he developed a desire for a college education is not told us, but clearly he possessed a very keen motive for it. Such was his zeal that he prepared for college within 12 months, although apparently he had not studied languages before. For this reason he was ill-prepared in all except mathematics when he entered Harvard College in 1816.

His life in college and after.—Throughout his college course he was recognized as excelling all his classmates in mathematics. It is said that he applied himself with equal faithfulness to the classics, and in spite of his poor preparation he commanded the respect of his instructors and stood well in these classes. In mathematics he mastered the calculus and read through a considerable portion of the great work of Laplace. He graduated in August, 1820, at the age of 27.

Colburn received the genuine respect of all who knew him. At college he was liked and respected by all his classmates, although he was not accustomed to participate in the social activities of college life. He was older and more mature than his fellow students and seems to have taken his college studies very seriously. He was not brilliant in conversation nor in public speech. Soon after Colburn's death, Dr. Edward G. Davis, a classmate, wrote of him as follows:

> In the constitution of Colburn's mind, many circumstances were peculiar. His mental operations were not rapid, and it was only with great patience and long-continued thought that he achieved his objects. This peculiarity, which was joined with an uncommon power of abstraction, he possessed in common with some of the most gifted minds which the world has produced. Newton himself, said that it was only by patient reflection that he arrived at his great results, and not by sudden or rapid flights. In Colburn this slowness and patience of investigation were leading traits. It was not his habit, perhaps not within his power, to arrive at rapid conclusions on any subject. * * * His conclusions, reached slowly and painfully, were established on a solid basis, and the silent progress of time, that great test of truth, has served but to verify and confirm them.[1]

The trend of his mind at the time of leaving college is reflected in his thesis, which was "On the Benefit Accruing to an Individual from a Knowledge of the Physical Sciences." One paragraph is especially significant:

> The purpose of education is to render a man happy as an individual, and agreeable, useful, and respectable as a member of society. To do this, he ought to cultivate all the powers of his mind, and endeavor to acquire a general knowledge of every department of literature and science, and a general acquaintance with the world by habits of conversation. And this is not inconsistent with the most intense application to a favorite pursuit.[2]

His life after leaving college is an example of the opinion he expresses here. Although engaged as a superintendent of a man-

[1] Barnard's Amer. Jour. of Educ., 2: 297. [2] Ibid., 300.

ufacturing company, a position of responsibility and one which required constant attention, Colburn found time to continue his educational endeavors. His algebra was completed after leaving the schoolroom. He was one of the founders of the American Institute of Instruction, and in 1830 delivered a masterly address before that body on "The Teaching of Arithmetic."

During his collegiate course he taught during the winter months in Boston, in Leominster, and in Canton. After leaving college he began teaching in a select school in Boston. He continued in this school for about two and a half years. He then gave up school-teaching and went to Waltham as superintendent of the Boston Manufacturing Co. In August, 1824, he became superintendent of the Lowell Merrimack Manufacturing Co., at Lowell. He continued in this position until his death, September 13, 1833.

In the winter of 1826, Lowell was incorporated as a town, and at the first town meeting Mr. Colburn was chosen a member of the superintending school committee. It is said that in order to provide time for proper attention to the affairs of the new school system, the committee often held their meeting at 6 o'clock in the morning. Mr. Colburn served on this committee for two years and was reelected in 1831, but was excused at his own request. He was elected a fellow of the American Academy of Arts and Science in 1827. Also, for a number of years he was a member of the examining committee for mathematics at Harvard College.

While at Lowell he conceived a scheme for the intellectual improvement of the community by popular lectures on scientific subjects. Throughout the autumn and winter of 1825 he gave illustrated lectures upon natural history, light, the seasons, and electricity. He continued giving popular lectures for several years, varying the content somewhat from year to year. On one occasion he received and accepted an invitation to deliver a series of lectures before the Mechanics Charitable Association in Boston.

Colburn's arithmetics.—It was while teaching in Boston that he wrote his arithmetics. The *First Lessons in Intellectual Arithmetic* came from the press in the autumn of 1821. The *Sequel to the First Lessons* was published about a year later.[1]

In 1825 Colburn published *An Introduction to Algebra upon the Inductive Method of Instruction.* Although the algebra was not published until after he had ceased teaching, it was a part of his originally conceived plan which had its incipiency in his teaching experience.

[1] The date of publication of the Sequel has been erroneously given as 1824, and one writer has given it as 1826. Neither of these dates is correct. The compiler of this report has in his possession a copy bearing the date of 1822, the date of copyright being October 30.

Mr. Batchelder, of Cambridge, states:

I remember once, in conversing with him with respect to his arithmetic (the First Lessons), he remarked that the pupils who were under his tuition made his arithmetic for him; that he had only to give his attention to the questions they asked, and the proper answers and explanations to be given, in order to anticipate the doubts and difficulties that would arise in the minds of other pupils; and the removal of these doubts and difficulties in the simplest manner was the foundation of that system of instruction which his schoolbooks were the means of introducing.[1]

He published about the same time a series of reading books for young children. Each book of the series contained some appropriate instructions in English grammar. It is said that his method of presenting grammar gave results scarcely less admirable than in arithmetic. Before his death he had planned a revision of his Sequel which was intended to meet the criticisms which had been made upon it. Unfortunately he had not committed his ideas for the revision to writing and nothing has come down to us of what probably would have been a work of even more merit.

The First Lessons was immediately introduced into the schools and enjoyed greater popularity than any other arithmetic ever published. In 1856 the statement was made that 50,000 copies were used annually in Great Britain and 100,000 annually in the United States. It was even translated into foreign languages. In the Boston Public Library there is a copy printed in raised type for the blind.

The first teachers who used the First Lessons were enthusiastic in their praise of it. Mr. Thomas Sherwin, principal of the high school, Boston, said:

I regard Mr. Colburn as the greatest benefactor of his age, with respect to the proper development of the mathematical powers. Pestalozzi, indeed, first conceived the plan; but Mr. Colburn realized the plan, popularized it, and rendered it capable of being applied by the humblest mediocrity. Indeed, I regard the First Lessons as the *ne plus ultra* of primary arithmetic.[2]

Speaking of the First Lessons, Mr. Page said in 1843, in addressing the American Institute of Instruction:

The reason, the understanding, is addressed, and led on step by step, till the whole is taken into the mind and becomes a part of it. The memory is little thought of, yet the memory can not let it slip; for what has been drunk in, as it were, by the understanding, and made a part of the mind, the mind never forgets. To how many a wayworn and weary pupil under the old system; to how many a proficient who could number his half dozen authors, and twice that number of manuscript cyphering books; to how many a teacher even, who had taught the old system, winter after winter, and yet saw but as "through a glass darkly"; to how many such, was this book on its appearance Their First Lessons in Arithmetic? Warren Colburn's name should be written in a conspicuous place, in letters of gold, for this service.[3]

[1] Ibid., p. 300. [2] Ibid., p. 309.

[3] From a lecture before the American Institute of Instruction, August, 1843.

Pestalozzi's contribution to arithmetic.—By stating in the title of his first text that it was "on the plan of Pestalozzi" and by the use of the Pestalozzian unit table and fraction tables, Colburn definitely connected his work with that of Pestalozzi, the noted Swiss educator. Statements made by writers [1] in recent years have tended to create the general impression that Colburn merely copied what Pestalozzi had already done. In presenting the evidence on this point, we shall first review certain phases of Pestalozzi's work.

Johann Heinrich Pestalozzi (1746–1827) was profoundly influenced by the writings of Rousseau, particularly by the Emile. Early in life he conceived the ambition of improving the social, moral, and religious conditions which then existed among the peasant class of Europe. This was the guiding purpose of his life. At first he strove to accomplish it by reforming the vagrant and delinquent children by means of industrial training, but after his work at Stanz in 1799, where the conditions made industrial training impossible, he directed his attention to improving elementary instruction, which he concluded was a more fundamental means [2] of social improvement. This conclusion was based upon the assumption that one's social, moral, and religious status was dependent upon one's ability to form "clear ideas" from confused sense-impressions and that the ability to form such ideas was a general ability and capable of development as such.[3]

Pestalozzi states his concept of the immediate purpose of instruction when he says "in the youngest years we must not reason with children, but must limit ourselves to the means of developing their minds." [4] His meaning is made more clear when he says in another place:

> The world lies before our eyes like a sea of confused sense-impressions, flowing one into the other. If our development through nature only is not sufficiently rapid and unimpeded, the business of instruction is to remove the confusion of these sense-impressions; to separate the objects one from another; to put together in imagination those that resemble or are related to each other; and in this way to make all clear to us, and by perfect clearness in these to raise in us distinct ideas.[5]

According to Pestalozzi nature has endowed the child with instincts and capacities or "faculties." These develop naturally, but this process is too slow, and man is to assist in the development of the child's "faculties" by using the same materials and art of instruction that are employed by nature. "The means of making clear all knowledge gained by sense-impressions," he says, "comes from *number*, *form*, and *language*." These are the "elementary means of

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[1] F. P. Graves; A History of Education in Modern Times, p. 151.
[2] How Gertrude Teaches Her Children, p. 227.
[3] Ibid., p. 145.
[4] Ibid., p. 67.
[5] Ibid., p. 142.

instruction, because the whole sum of external properties of any object is comprised in its outline and its numbers, and is brought home to my consciousness through language." This thesis furnished the basis for his curriculum. The method of instruction was based upon the thesis that sense-impressions are the "absolute foundation of all knowledge."

Pestalozzi considered arithmetic the most important means for giving that mental training which would result in the power to form "clear ideas."[1] For this reason he took particular pains to identify the elements of the subject, and to formulate the series of steps in the method of instruction. He considered arithmetic to arise—

entirely from simply putting together and separating several units. Its basis is essentially this: *One and one are two, and one from two leaves one.* Any number, whatever it may be, is only an abbreviation of this natural, original method of counting. But it is important that this consciousness of the origin of relations of numbers should not be weakened in the human mind by the shortening expedients of arithmetic. It should be deeply impressed with great care on all the ways in which this art is taught; and all future steps should be built upon the consciousness, deeply retained in the human mind, of the real relations of things which lie at the bottom of all calculation. If this is not done, this first means of gaining clear ideas will be degraded to a plaything of our memory and imagination, and will be useless for its essential purpose.[2]

Both the content of arithmetic and the method of teaching it, as Pestalozzi conceived them, are implied in this statement. The "clear idea" which is represented by a number, e. g., seven, is obtained by *counting* seven objects. The "clear idea" which is represented by 7 multiplied by 8 is to be obtained by *counting* the total number of objects in seven groups containing eight objects each. This plan was extended to common fractions and operations with them. The "shortening expedients of arithmetic" were not permitted until "clear ideas" of numbers and number relations had become permanently fixed by having appropriate sense perceptions. In the beginning, the children might use their fingers, peas, stones, or other handy objects for obtaining the necessary sense perceptions. Later a "spelling board" with moveable letters (tablets) was used or the tables which Pestalozzi devised.

The units table consisted of 100[3] rectangles arranged in rows of 10. Each rectangle in the first row contained 1 vertical mark. Each rectangle in the second row contained 2 vertical marks, and so on, each rectangle in the tenth row containing 10 vertical marks. The first fraction table was made up of 10 rows of squares, each row con-

[1] How Gertrude Teaches Her Children, p. 209.

[2] Ibid., pp. 210-211.

[3] In How Gertrude Teaches Her Children, the translator (p. 216) states that the units table consisted of 12 rows of 12 rectangles each and that each of the fraction tables contained 144 squares. The tables are described by Unger, p. 177, as having only 10 rows, each consisting of 10 rectangles, or squares. The tables were reproduced by Colburn in this form. This form is also given in Pestalozzi's Ausgewählte Werke, by F. Mann.

taining 10 squares. The squares of the first row were undivided. Those in the second were divided into two equal parts by a vertical line: those of the third into three equal parts, and so on. The second fraction table was constructed from the first by dividing the squares in the second column into two equal parts by a horizontal line, those of the third column into three equal parts, and so on.

These tables were used in an elaborate set of exercises which were based upon Pestalozzi's concept of the nature of arithmetic and of the art of instruction. The exercises were prepared by Hermann Krüsi, a teacher of experience and ability who was an assistant to Pestalozzi at Burgdorf and later at Yverdun. They were published in 1803 with the title, *Anschauungslehre der Zahlenverhältnisse*, in three parts.[1]

There were eight sets of exercises upon the units table.[2] In the first, the child was to point to the marks in the table and count out the combinations of the multiplication table up to 10 times 10. The second consisted of 540 exercises of the form: "19 times 1 is 9 times 2 and 1 time the half of 2." The object was to express each number as so many twos, threes, fours, etc. In the third, a number expressed in terms of sixes was changed to so many sevens, or if expressed in terms of sevens, it was changed to so many eights, etc. For example, "9 times 9 and 8 times the ninth part of 9 is 89 times 1, 89 times 1 is 8 times 10 and 9 times the tenth part of 10." In the fourth, the tenth parts of numbers are multiplied by the numbers 1 to 10. The remaining sets of exercises were made increasingly complex, the sixth consisting of 360 exercises of the form, "12 is 2 times 6, 18 is 3 times 6, therefore 2 times 6 is 2 times the third part of 3 times 6." Four of the eight sets of exercises contained a total of more than 2,000 exercises of the formal types illustrated.

The first fraction table was made the basis of 12 sets of exercises and the second of 8 more. One of these contained 17,280 exercises of the form "17 halves are 2 times 7 halves and 3 times the seventh part of 7 halves."

These exercises are purely formal, but Pestalozzi believed that by having a child grind through them laboriously, counting out each on the appropriate table, his mental powers would be developed, because the exercises were based upon his psychological analysis of the process of the development of the human mind. Arithmetic had been reduced to its elements and the instruction psychologized by reduction to an elaborate formula.

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[1] There is a copy of *Anschauungslehre der Zahlenverhältnisse* in the Library of Congress. This copy contains only the first part, which was devoted to the exercises on the units table.

[2] The facts of this description are taken from *Die Methodik der Praktischen Arithmetik*, by Friedrich Unger, p. 177ff.

It should be noted that no practical problems are included in the list, and there is no suggestion that arithmetic has a practical function. In the first stages, the child was expected to count familiar objects. Some years earlier Pestalozzi said in "Leonard and Gertrude":

> The instruction she [Gertrude] gave them in the rudiments of arithmetic was intimately connected with the realities of life. She taught them to count the number of steps from one end of the room to the other; and two rows of five panes each, in one of the windows, gave her an opportunity to unfold the decimal relations of numbers. She also made them count their threads while spinning, and the number of turns on the reel when they wound the yarn into skeins.[1]

No problems are involved here. The children were *made* to count these objects. But at this time Pestalozzi had not begun to formulate the art of instruction, and when he did, the idea suggested in this quotation was overshadowed by his interest in a psychological method. Since "clear ideas" of numbers and their relations were of the first importance, the symbols of arithmetic were to be delayed until the clear ideas were fixed in the mind of the child. The forms of operations were not included in the published plan. Thus the instruction necessarily became oral.

No better summary of Pestalozzi's system of arithmetic can be given than that found in Biber's *Henry Pestalozzi and His Plan of Education*, which was published in 1831. He says:

> In this calculating world shall we be understood if we say that Pestalozzi's arithmetic had no reference to the shop or counting house; that it dealt not in monies, weights, or measures; that its interests consisted entirely in the mental exercise which it involved and its benefit in the increase of strength and acuteness which the mind derived from that exercise?
>
> Again, in this mechanical sign-loving age, shall we be understood if we say that his arithmetic was not the art of handling and pronouncing ciphers, but the power of comprehending and comparing numbers? And, lastly, with a public whose faith is exclusively devoted to what is immediately and palpably "practical and useful," shall we be believed if we add that, notwithstanding the apparently unpractical tendency of Pestalozzi's arithmetical instruction, he numbered among his pupils the most acute and rapid "practical arithmeticians"?
>
> Such, however, was the case; his course of arithmetic excluded ciphers until numbers were perfectly understood, and the rules of reduction, exchange, and others, in which arithmetic is applied to the common business of life, were superadded at the close of his arithmetical course, as the pupil's future calling might require it. The main object of his instruction in this branch of knowledge was the development of the mental powers; and this he accomplished with so much success that the ability which pupils displayed, especially in mental arithmetic, was the chief means of attracting the public notice to his experiments.

The Pestalozzian movement in America.—William McClure, a Scotch philanthropist, was the first disciple of Pestalozzi in the United States. The earliest presentation of Pestalozzian principles was by him in an article published in the *National Intelligencer*, June 6, 1806.[2]

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[1] Pp. 130-131.

[2] Will S. Monroe: History of the Pestalozzian Movement in the United States, p. 44.

This was followed by other articles of a more elaborate nature. In 1806 McClure induced Joseph Neef, who had worked with Pestalozzi, to come to Philadelphia, where he opened a Pestalozzian school in 1809. About three years later Neef removed to Village Green, Pa. From there he removed to Louisville, Ky.; thence to New Harmony, Ind.; and finally to Cincinnati. In 1808 he published a treatise on education, entitled: *Sketch of a plan and method of education founded on the analysis of the human faculties and natural reason, fitted for the offspring of a free people and of all rational beings.* A chapter was devoted to Pestalozzi's plan of teaching arithmetic.

The work of Neef and the writings of McClure served to advertise the principles of Pestalozzi in the United States, but educational practice was not influenced directly. This was particularly true in New England before 1821. There were leaders in education who were acquainted with the work of Pestalozzi, and a little later there were many enthusiastic disciples of the Swiss schoolmaster. Educational periodicals, beginning with the Academician (1818–19), contained many articles on the work of Pestalozzi. In 1821, when Warren Colburn published his *First Lessons in Arithmetic on the Plan of Pestalozzi*, the Pestalozzian movement in the United States was beginning to acquire momentum and to influence school practice.

Colburn's relation to Pestalozzi.—In the preface to the first editions [1] of the First Lessons, Colburn acknowledges his indebtedness to Pestalozzi as follows:

> In forming and arranging the several combinations the author has received considerable assistance from the system of Pestalozzi. He has not, however, had an opportunity of seeing Pestalozzi's own work on this subject, but only a brief outline of it by another. The plates also are from Pestalozzi. In selecting and arranging the examples to illustrate these combinations, and in the manner of solving questions generally, he has received no assistance from Pestalozzi.

The meaning of this statement becomes clear only when we consider the meaning which Colburn attached to the words, "combinations," and "examples to illustrate these combinations." The "several combinations" to which Colburn refers are the number facts such as, "Eight and four are how many?," "Three times seven are how many?," "Fourteen less nine are how many?" "Eight are how many times six?" "6 is one-fifth of what number?" The "examples" are practical problems about things which children can understand. It appears that even in the early editions of the First Lessons, the Pestalozzian tables, or plates as Colburn called them, did not always accompany the text. Colburn, in his address on "The Teaching of Arithmetic," 1830, said: "It has often been asked whether the plates which sometimes accompany Colburn's *Intel-*

[1] This acknowledgment does not appear in the first edition, 1821, but does appear in the editions of 1822 and 1826.

lectual Arithmetic, or anything else of a similar nature, are of any use to the learner."

In addition to this indebtedness to Pestalozzi, which Colburn explicitly acknowledges, a study of his writings shows that some of the underlying principles of his texts are essentially identical with those held by Pestalozzi. This probably means that Colburn was acquainted with Pestalozzi's principles. But to appreciate fully the extent of Colburn's indebtedness to Pestalozzi, it is necessary to consider what he contributed as well as what he borrowed, and how critically he selected what he used.

Soon after Colburn's death, Dr. Edward G. Davis, to whom reference has already been made on page 54, wrote as follows:

His great and most interesting project, that of improving the system of elementary instruction in mathematical science, appears to have occurred to him during the latter part of his college life, and was the subject of painful thought many years before his first work made its appearance. It required, indeed, no small energy of mind thus to break through the trammels of early education, and strike out a new path; for Colburn, like others, had been brought up under a system the reverse of that which he now undertook to mature and introduce.[1]

Colburn's biographer says:

His First Lessons was, unquestionably, the result of his own teaching. He made the book because he needed it, and because such a book was needed in the community. He had read Pestalozzi, probably, while in college. That which suited his taste, that which he deemed practicable and important, he imbibed and made his own. He has been sometimes represented as owing his fame to Pestalozzi. That in reading the account and writings of the Swiss philosopher he derived aid and confidence in his own investigations of the general principles of education, is true. But, his indebtedness to Pestalozzi is believed to have been misunderstood and overrated.[2]

After examining carefully all of the evidence which has been obtainable, it is scarcely possible to improve upon the justness of these estimates of the originality of Warren Colburn's work. He died at the age of 40. This, coupled with the fact that he did not begin to prepare for college until he was in his twenty-third year, and that he taught school only two and a half years after graduating from Harvard, indicates the genius of the man. He had the ability and courage "to break through the trammels" of tradition and of his own education. With only slight assistance from the work of Pestalozzi, Colburn produced a text which revolutionized our school practice as no other text has done.

[1] Barnard's Amer. Jour. of Educ., 2: 297. [2] Ibid., p. 301.

Chapter V.

WARREN COLBURN'S ARITHMETICS.

The function of arithmetic.—Colburn recognized as of prime importance the utilitarian value of arithmetic, but he accorded an almost equal value to the subject as a "discipline of the mind." He says:

Arithmetic, when properly taught, is acknowledged by all to be very important as a discipline of the mind; so much so that, even if it had no practical application which should render it valuable on its own account, it would still be well worth while to bestow a considerable portion of time on it for this purpose alone. This is a very important consideration, though a secondary one compared with its practical utility.[1]

Also, in another place:

Few exercises strengthen and mature the mind so much as arithmetical calculations, if the examples are made sufficiently simple to be understood by the pupil, because a regular, though simple, process of reasoning is requisite to perform them, and the results are attended with certainty.[2]

Colburn emphasized arithmetic as a factor of the child's education, and he desired that it be taught to children 5 or 6 years of age:

The fondness which children usually manifest for these exercises, and the facility with which they perform them, seem to indicate that the science of numbers, to a certain extent, should be among the first lessons taught them.[2]

The First Lessons were intended to be begun at the age of 5 or 6 and studied for three or four years, and then the pupil was to advance to the Sequel.

The ability to decide what operations were demanded by arithmetical situations was emphasized. The absence of rules, the emphasis upon the mental processes, allowing the pupil to do the example his own way at first and think his way through it, all are representative of Colburn's attitude. Colburn sought to make the pupil resourceful. He also wished to make the pupil skillful in performing the operations. This is shown by the large number of drill problems. Of the first 1,000 problems, 75 per cent are abstract and for drill.

THE FIRST LESSONS.

The arithmetic which became known as *Colburn's First Lessons* was first published at Boston in 1821, with the title, *First Lessons in Arithmetic, on the plan of Pestalozzi. With some improvements.*[3] In

[1] Address, "The Teaching of Arithmetic."

[2] Preface to First Lessons.

[3] There is a copy of the first edition in the library of the American Antiquarian Society at Worcester, Mass. The second edition, 1822, had the same title.

1826 it had the title, *Colburn's First Lessons. Intellectual Arithmetic upon the Inductive Method of Instruction*, which it still retains. The second edition, 1822, contains only a few important changes from the first edition, although the number of pages was increased from 108 to 172. In the first edition only the first three sections, which contain the four fundamental operations, were commenced with practical examples. In the second edition "every combination is commenced with practical examples." Since 1822 the body of the text has remained unchanged, except in the revised edition of 1884, of which we shall speak below. The Pestalozzian tables accompanied[1] the editions of 1821, 1822, and 1826, which have been examined in the preparation of this report, but there is no reference to them in the editions of 1847 and later. In 1830 Colburn spoke of the tables as *sometimes* accompanying the text. The edition of 1847 contains directions for eight preliminary lessons in which the pupils are to be taught by counting objects. Only a small part of the original preface appears in this edition.

In June, 1863, Mr. Henry O. Houghton took over the First Lessons; the original preface was restored; and Part III, of 11 pages on "Written Arithmetic," was added. Otherwise this edition of the text is essentially identical with the edition of 1826, except no reference is made to the plates which accompanied that edition. A revised edition was published in 1884. The book was thoroughly revised and enlarged, many of the prominent features of the earlier edition being entirely lost. Written arithmetic is mixed with the mental, the Hindu notation is introduced earlier, the numbers 2, 3, 4, etc., are taken up formally and separately (a tendency toward the Grube method, q. v.), and illustrations (pictures) are used.

These two editions of Colburn's First Lessons are still published by Houghton Mifflin Co., who assured an inquirer in 1913 that they were "still actively in print." The sale of the book has declined in recent years, but several thousand copies are still used annually. The company mentioned a recent single order for 1,000 copies. Its use extends from New England to the Western States.

Colburn's arithmetic, which is commonly spoken of as the "Sequel," was first published in 1822, with the title, *Arithmetic; Being a Sequel to First Lessons in Arithmetic*. The Sequel has no such interesting history as the First Lessons. The original form was not revised. While it enjoyed a fair degree of popularity, it was small compared with that of the First Lessons. Editions were printed in 1841, 1849, and as late as 1860.

Plan of the First Lessons.[2]—The book itself is divided into two parts. The first contains the examples, the tables of the common

[1] The tables do not appear to have been bound with the text, but separately in a small pamphlet.
[2] Unless otherwise stated, the description of Colburn's First Lessons will be based upon the 1826 edition.

denominate numbers, the systems of notation up to 100, and a few explanatory notes. Part II is called a Key, and is primarily for the use of the teacher. In the earlier editions it included "an explanation of the plates," the manner of using them, and the solutions of the "most difficult of the practical examples." These practical examples were solved in such a way as to "show the principles by which they are performed." In the later editions the portion which explained the plates was omitted.

The primary purposes of the book were to furnish the child with practical examples which required arithmetical operations and to provide exercises for drill upon the combinations which the child discovers are needed to solve the examples proposed. With few exceptions the practical examples are taken from situations in the life of children or from situations which children easily understand. The examples are about buying oranges, dividing apples among playmates, buying family provisions, counting marbles, etc. There are a few examples from situations in commerce, but on the whole the problems of the text stand out in marked contrast to the commercial problems with which the texts of the previous period were filled. In addition to the practical examples, there are well-graded lists of abstract exercises for drill. They stand to the practical examples in about the ratio of three to one.

The contents of the First Lessons.—Section I, which covers 19 pages, is concerned with addition and subtraction. Neither the addition nor the subtraction table is given. The first article consists of very simple "practical questions," and in the second article the addition facts are called for in regular order by questions such as: "Two and one are how many?"; "Two and two are how many?", etc. In the third article the same questions are repeated, but the order is varied. The answers to the questions are not given in the book. Colburn assumes that the pupil has grasped the idea of addition from the practical questions of the first article. Knowing the meaning of such questions as "Three and two are how many?" the pupil can easily find the answer for himself. In the process of discovery he is to use sensible objects, such as beans, nuts, etc., or the plates.

The next article has to do with larger numbers, and in some instances there are three or more numbers to be added together. The numbers from 1 to 10 are to be added to the numbers from 10 to 20. In the fifth article subtraction is treated briefly, and in the next the numbers 1 to 10 are added to the numbers 20 to 100. All the preceding are then combined together, and the section closes with a list of "practical questions which show the application of all the preceding articles."

Multiplication is introduced in Section II with such examples as: "What cost three yards of tape, at two cents a yard?"; "What cost four apples at two cents apiece?" Colburn remarks that the pupil will see no difference between this and addition, and it is best he should not at first. After a while the idea of multiplication is to be explained to the pupil, and he is to give the solution to the problem in this form: "If one yard cost two cents, three yards will cost three times two cents." The multiplication table is treated in the same manner as the addition table.

Division is begun with examples such as: "How many pears, at two cents apiece, can you buy for four cents?" After five examples of this sort, the pupil is asked, "If you have eight apples to give to four boys, how many can you give to each?" The pupil is not told that this example involves division, but it is expected that "the pupil will scarcely distinguish it from multiplication." He is to solve the example by using his knowledge of the multiplication table or by counting it out with objects.

In the second article of Section III, common fractions are introduced. One-half is defined in a remark as one of two equal parts of a thing or number. Later a third and a fourth are defined in the same way. The pupil is given such questions as, "If an apple is worth two cents, what is one-half of it worth?" "What is one-half of two cents?" This last is answered, and the question "Why?" is asked. The answer to this is given, "Because if you divide two cents into two equal parts, one of the parts is one cent."

In general, no answers and no suggestions are given to the practical examples in the text because they are "so arranged that the *names* will usually show the pupil how the operation is to be performed." But in the case of abstract examples, answers are frequently given in this section.

The common fractions from fourths to tenths are not explained, but they are taken up in order and the pupil is drilled on them in this fashion:

When wheat is eight shillings a bushel, what is one-eighth of a bushel worth? What are two-eighths of a bushel worth? What are three-eighths of a bushel worth? What are four-eighths of a bushel worth? Five-eighths? Six-eighths? Seven-eighths?

When wood is eight dollars a cord, what part of a cord can you buy for a dollar? What part of a cord can you buy for two dollars? What part for three dollars? What part for four dollars? What part for five dollars? What part for six dollars? What part for seven dollars? How much can you buy for nine dollars? How much for ten dollars? How much for eleven dollars? How much for thirteen dollars? How much for fifteen dollars? How much for nineteen dollars?

What part of eight is one?
What part of eight is two?
Three is what part of eight?
Four is what part of eight?

Five is what part of eight?
What do you understand by one-eighth, two-eighths, etc., of any number?
Seven is what part of eight?
How many eighths make a whole one?
Ten are how many times eight?
Eleven are how many times eight?
Twelve are how many times eight?
Thirteen are how many times eight?
Fourteen are how many times eight?

Section IV contains such problems as:

If a yard of cloth cost 4 dollars, what will 5 yards and 3 fourths of a yard cost? A man bought 2 oranges at 6 cents apiece: how many cents do they come to? He paid for them with cherries at 4 cents a pint; how many pints did it take? James had 8 oranges that were worth 5 cents apiece, and George had 5 quarts of cherries that were worth 6 cents a quart, which he gave to James for a part of his oranges. How many oranges did he buy, and how many had James left?

Problems like this last will be recognized as coming under the head of barter. (See p. 29.) Colburn leads up to them by problems like the second above, but no rule or suggestion is given. In problems like the first the operation calls for the multiplication of a mixed number by an integer.

Section V is devoted to such practical examples as the following, and to abstract examples to exercise the pupil upon the combinations required:

James had 4 apples and John had half as many; how many had he? If 3 barrels of cider cost 9 dollars, what part of 9 dollars will 1 barrel cost? What part of 9 dollars will 2 barrels cost? A boy having 12 apples kept 1 fourth of them himself and divided the other 3 fourths of them equally among 4 of his companions; how many did he give them apiece? If 2 men can do a piece of work in 6 days, how long would it take 4 men to do the same work?

In the next section the pupil is asked to find the whole when a part is given. Some of the more difficult problems are:

A man sold a cow for 21 dollars, which was only seven-tenths of what she cost him; how much did she cost him? When he bought her, he paid for her with cloth at 8 dollars a yard; how many yards of cloth did he give?

There is a school in which 2 ninths of the boys learn arithmetic, 3 ninths learn grammar, 1 ninth learn geography, and 12 learn to write. How many are there in the school, and how many attending to each study?

The section closes with a miscellaneous list in which there are such problems as:

A fox is 80 rods before a greyhound and is running at the rate of 27 rods in a minute; the greyhound is following at the rate of 31 rods a minute; in how many minutes will the greyhound overtake the fox?

If a staff 3 feet long cast a shadow of 2 feet at 12 o'clock, what is the length of a pole that casts a shadow 18 feet at the same time of day?

Section VII contains exercises for drill upon the multiplication table for the numbers 10 to 20 multiplied by the first 10 numbers, together with some complications of the preceding combinations. It is suggested that this section may be omitted until reviewing the book. The next four sections contain questions which call for special cases of the division of an integer or a mixed number by a fraction, as; "How many one-thirds in 4?" or "How many thirds in two and one-third?" and for the multiplication of fractions and mixed numbers by integers in general.

In Section XII the symbolism of fractions is given for the first time, and exercises upon the combinations of the preceding four sections are stated in terms of fractional symbolism. In the following sections these operations are taken up in the order named: Reduction of fractions to a common denominator, addition and subtraction of fractions, reduction of fractions to lowest terms, division of fractions by whole numbers, multiplication of one fraction by another, division of whole numbers by fractions, and division of one fraction by another.

Colburn remarks that the division of a fraction by a whole number calls for the same operation as the multiplication of a fraction by a fraction. For this reason he places together the problems which require these combinations. He also points out that it is difficult to find problems which require a fraction to be reduced to its lowest terms. For this operation he gives only abstract examples, but he suggests that it would be well to omit this article the first time the pupil goes through the book, and "after he has seen the use of the operation let him study it."

The tables of Federal money, sterling money, troy weight, avoirdupois weight, cloth measure, wine measure, dry measure, the measure of time, and a list of 183 miscellaneous problems are given as a sort of an appendix.

In such problems as the following the notion of rate is expressed:

A man failing in trade was able to pay his creditors only 4 shillings on a dollar; how much would he pay on 2 dollars? How much on 3 dollars? How much on 7 dollars? How much on 10 dollars?

Interest is introduced with a note which explains the meaning of the term. After explaining that "6 per cent" means 6 cents on a dollar, 6 dollars on a hundred dollars, or 6 pounds on a hundred pounds, he makes the generalization that it is "6/100 of the sum, whatever the denomination." The pupil is given such problems as:

The interest of 1 dollar being 6 cents for 1 year, what is the interest of 7 dollars for the same time? What is the interest of 10 dollars? Of 15 dollars? Of 20 dollars? Of 30 dollars? Of 50 dollars? Of 75 dollars? Of 100 dollars? Of 118 dollars?

Finally, the pupil is given such as these to solve:

If the interest of 2 months or 60 days is 1 per cent, what would be the per cent for 20 days? What for 40 days? What for 15 days? What for 45 days? What for 12 days? What for 10 days? What for 5 days?

What is the interest of 100 and 37 dollars for 2 years 3 months and 20 days?

Fellowship is presented by such problems as:

Two men bought a bushel of corn, one gave 1 shilling, the other 2 shillings; what part of the whole did each pay? What part of the corn must each have?

Two men hired a pasture for 58 dollars; one put in 7 horses, and the other 3 horses; what ought each to pay?

Three men commenced to trade together; they put in money in the following proportion; the first, 3 dollars, as often as the second put in 4, and as often as the third put in 5; they gained 87 dollars. What was each man's share of the gain?

Two men hired a pasture for 32 dollars. The first put in 3 sheep for four months, the second put in 4 sheep for five months; how much ought each to pay?

Following this last problem, which is the first in double, or compound fellowship, an explanatory note of five lines is given. In the case of simple fellowship no explanation is given.

There are a few arithmetical puzzles of which the following are typical:

Said Harry to Dick, my purse and money together are worth 16 dollars, but the money is worth 7 times as much as the purse; how much money was there in the purse? and what is the value of the purse?

A man having a horse, a cow, and a sheep, was asked what was the value of each. He answered that the cow was worth twice as much as the sheep, and the horse 3 times as much as the sheep, and that all together were worth 60 dollars. What was the value of each?

A man driving his geese to market was met by another, who said, "Good morrow, master, with your hundred geese." Says he, "I have not a hundred, but if I had half as many more as I now have, and two geese and a half, I should have a hundred." How many had he?

What number is that, to which if its half and its third be added the sum will be 55?

Objective materials.—In the Key directions are given for using the Pestalozzian tables[1] and other objective materials. Before 1821, children used their fingers, and even their toes, in learning to count, and probably counted out problems on them. But this practice seems to have been tolerated rather than recognized as a legitimate and valuable method of learning number facts. Certain it is that Colburn was the first author in the United States to introduce objective materials in an arithmetic text. The plates represent just one type of objects which he used. Beans, grains of corn, pieces of crayon, marks, etc., are recommended for use and even preferred. He says:

The first examples may be solved by means of beans, peas, etc., or by Plate I. The *former* method is preferable, if the pupil be very young, not only for the examples in the first part of this section, but for the first examples in all the sections.[2]

Mental arithmetic.—Colburn's First Lessons is an "intellectual" arithmetic, i. e., the examples are to be solved without the use of pencil and slate or paper. The Hindu symbols for writing numbers are not given until page 50, and methods of calculating with figures are not given. Numbers greater than 100 occur in very few problems, but within this quantitative range Colburn has treated many

[1] See p. 57 for a description of these tables. [2] Key to First Lessons, p. 144.

of the topics which we have found in texts of the ciphering-book period. A comparison reveals the following: Notation, the four operations for integers, practically all of the important denominate numbers and the operations upon them, common fractions completely, rule of three, direct and inverse, barter, practice, single and double fellowship, and interest. The important omissions are decimal fractions, exchange, evolution, loss and gain, and alligation. The topics omitted have to do with situations with which young children are relatively unacquainted. Exchange, and loss and gain dealt with situations peculiar to a professional business man. Decimal fractions were tools of calculations of business or of evolution. Alligation was an obsolescent topic.

Summary.—Colburn says of the plan of this book that it "entirely supersedes the necessity of any rules, and the book contains none." The child is to be given a practical example and from his understanding of the situation involved he is to decide upon the operation or operations to be performed. If he can not do this when the numbers are made simple, Colburn says that he is not ready for such an example. Colburn held that abstract exercises were more suitable for reducing the combinations to the level of habit than practical examples. And it is this function which the abstract examples were intended to fulfill.

Upon completing Colburn's First Lessons, a pupil was acquainted with a large per cent of the quantitative situations which he would probably meet in life. He had met practical examples taken from these situations, and he had had to decide upon the combinations to be made. In this way he came to understand the situations so that he knew what combinations should be made, even though the quantities should be so large as to require written calculations. He had learned as much of notation and the symbolisms of arithmetic as he has needed. He knew the denominate quantities which he had met in the practical examples. And he had been thoroughly drilled upon the fundamental number facts.

THE SEQUEL.

As its title indicates, the Sequel was intended to be studied by the pupil after he had completed the First Lessons. Colburn states in the preface to the Sequel that the pupil may commence the First Lessons as soon as he can read the examples or perhaps even before. By doing this the pupil would be prepared to commence the Sequel by the time he was 8 or 9. It was written to be a practical arithmetic, but Colburn expected the pupil to learn something of the science of arithmetic as he worked with practical examples.

In his analysis of the subject matter of arithmetic, Colburn distinguished between the processes of arithmetic, which he calls "principles," and the application of arithmetic, which he designates as "subjects." To him the "principles" mean arithmetic and the applications merely a field for the exercise of these principles; denominate numbers, mensuration, percentage, interest, etc., are not taken as the basis for separate chapters, or even distinct topics. "To give the learner a knowledge of the principles" is his purpose, and to this end the problems are grouped about the principles.

Colburn takes the position that "When the principles are well understood, very few subjects will require a particular rule, and if the pupil is properly introduced to them, he will understand them better without a rule than with one." For example, if a pupil understands well the relation between a product and its factors in all its phases, percentage and its applications require no particular rule and will present no difficulty to the learner. At most the learner will need to be told the meaning of the new terms used in expressing the problem. As would naturally be expected from such a point of view, the applications of arithmetic do not influence nor determine the organization of Colburn's texts.

The plan of the Sequel.—The subdivisions and order of the "principles" are unusual. Multiplication of integers follows addition instead of subtraction. In fractions, multiplication is placed first and is followed by addition and subtraction. Both multiplication and division of fractions are divided into several cases. The Sequel is divided into two parts. The first consists of graded lists of problems with an occasional suggestive note to define some new term or to interpret the meaning of the problem. "The second part contains a development of the principles" based upon problems.

The two parts are to be studied together, when the pupil is old enough to comprehend the second part by reading it himself. When he has performed all the examples in an article in the first part, he should be required to recite the corresponding article in the second part, not verbatim, but to give a good account of the reasoning. When the principle is well understood, the rules [1] which are printed in italics should be committed to memory.

Colburn gives rules only for the principles and not for the applications of arithmetic. The table of contents of the Sequel makes no mention of any of the applications of arithmetic, several of which usually have a chapter devoted to them.

Colburn mentions the following "subjects" as being specifically included in the text: Compound multiplication, addition, subtraction and division; simple interest, commission, insurance, duties and premiums, common discount, compound interest, discount, barter, loss and gain, simple fellowship, compound fellowship, equation of payments, alligation medial, alligation alternate, square and cubic

[1] Few rules are given, and such as are given are placed at the end of a section. It is intended that the pupil will develop the rule as the result of solving problems before he reaches the printed statement.

measure, duodecimals, taxes, mensuration, geographical and astronomical questions, exchange, tables of denominate numbers.

Topics omitted.—Colburn omits some topics entirely. He specifically mentions the rule of three, position, and powers, and roots. The reasons he gives for their omission are:

Those who understand the principles sufficiently to comprehend the nature of the rule of three, can do much better without it than with it, for when used, it obscures rather than illustrates the subject to which it is applied.

* * * This (rule of position) is an artificial rule, the principle of which can not be well understood without the aid of algebra; and when algebra is understood, position is useless. Besides, all the examples which can be performed by position may be performed much more easily, and in a manner perfectly intelligible without it.

Powers and roots, though arithmetical operations, come more properly within the province of algebra.

It is interesting to note that some of the omissions which Colburn made nearly a century ago are still considered debatable by some teachers.

How the "principles" are presented.—A masterly exposition of our decimal system of numeration is given in which Colburn shows its function. After defining the numbers 1 to 10 as names given to collections of units, he continues:

In this manner we might continue to add units, and to give a name to each different collection. But it is easy to perceive that if it were continued to a great extent it would be absolutely impossible to remember the different names; and it would also be impossible to perform operations on large numbers. Besides, we must necessarily stop somewhere; and at whatever number we stop, it would still be possible to add more; and should we ever have occasion to do so, we should be obliged to invent new names for them, and to explain them to others. To avoid these inconveniences, a method has been contrived to express all the numbers that are necessary to be used, with very few names.

The first ten numbers have each a distinct name. The collection of ten simple units is then considered a unit; it is called a unit of the second order. We speak of the collections of ten, in the same manner that we speak of simple units; thus we say one ten, two tens, three tens, four tens, five tens, six tens, seven tens, eight tens, nine tens. These expressions are usually contracted; and instead of them we say ten, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety.

To express the numbers between the tens, the numbers below ten are to be added to the tens. Colburn then explains how the names of numbers which are used in common language have been derived by such a method. After telling how a hundred and a thousand are made up he indicates how "this principle may be continued to any extent," and expresses his admiration of the decimal system of numeration by saying:

Hence it appears that a very few names serve to express all the different numbers which we ever have occasion to use. To express all the numbers from one to nine thousand, nine hundred and ninety-nine, requires, properly speaking, but "twelve" different names. It will be shown hereafter that these twelve names express the numbers a great deal further.

The "Arabic" and Roman methods of writing numbers are carefully explained in 11 pages. The Roman system is given in a footnote, with the statement that "a short description of it may be interesting to some." In Part I it is not mentioned.

Although it was Colburn's plan that the pupil should study the First Lessons before commencing the Sequel, yet he wrote the Sequel in such a way that this would not be "absolutely necessary." For example, in the development of addition he begins with a problem any child who is old enough to study the book can understand. After defining addition as putting together two or more numbers to "ascertain what numbers they will form," he gives the problem: "A person bought an orange for 5 cents and a pear for 3 cents; how many cents did he pay for both?" This problem is solved by taking the 5 and joining the 3 "to it a single unit at a time."

Says Colburn:

A child is obliged to go through the process of adding units every time he has occasion to put two numbers together until he can remember the results. This, however, he soon learns to do if he has frequent occasions to put numbers together.

He also points out that the child can not make much progress in arithmetic until he learns perfectly the addition tables up to ten.

Colburn's development of carrying in addition is based upon the decimal structure of the system of numeration. The first practical example calls for 24 and 8 to be added. He points out that 24 is simply an abbreviation for 20 and 4. "To add eight to twenty-four, add eight and four, which are twelve. To twelve add twenty. But twelve is the same as ten and two, therefore we may say twenty and ten are thirty and two are thirty-two." Three more practical examples, each one becoming more difficult, are explained in the same way. He then defines "carrying" by saying: "The reserving of the tens, hundreds, etc., and adding them with other tens, hundreds, etc., is called carrying." The principle of carrying is further illustrated by the following example, whose solution he explains:

A merchant had all his money in bills of the following description, one-dollar bills, ten-dollar bills, hundred-dollar bills, thousand-dollar bills, etc.; each kind he kept in a separate box. Another merchant presented three notes for payment, one 2,673 dollars, another 849 dollars, and another 756 dollars. How much was the amount of all the notes; and how many bills of each sort did he pay, supposing he paid it with the least possible number of bills?

Additional illustration of the principle of carrying is given by writing the addends in this form: $4000+600+70+3$. And finally when he is ready to state the rule, Colburn says: "From what has been said, it appears that the operation of addition may be reduced to the following rule."

Multiplication immediately follows addition and is begun with this example: "How much will 4 gallons of molasses come to at 34 cents a gallon?" After the example is solved by addition, it is pointed out that "if it were required to find the price of 20, 30, or 100 gallons, the operation would become laborious." Colburn goes on to say:

> If I have learned that 4 times 4 are 16, and that 4 times 3 are 12, it is plain that I need not write the number 34 but once, and then I may say 4 times 4 are 16, reserving the 10 and writing the 6 units as in addition. Then again, 4 times 3 (tens) are 12 (tens) and 1 (ten which I reserved) are 13 (tens).

Multiplication is then defined as "addition performed in this manner."

Subtraction follows multiplication and is presented as the reverse of addition. Colburn begins by giving five examples which, "though apparently different," all require the same operation—i. e., subtraction. The pupil solves the first examples by using his knowledge of addition.

The operation for the case which requires "borrowing" is presented by writing the numbers thus: $\begin{array}{r} 62 \\ 17 \\ \hline \end{array}$ is written $\begin{array}{r} 50 + 12 \\ 10 + 7 \\ \hline \end{array}$

Division was considered to be a particularly difficult topic. Colburn starts with some simple problems which he handles in the following fashion:

> A boy having 32 apples wished to divide them equally among 8 of his companions. How many must he give them apiece?
>
> If the boy were not accustomed to calculating, he would probably divide them by giving one to each of the boys, and then another, and so on. But to give them one apiece would take 8 apples, and one apiece would take 8 more, and so on. The question then is, to see how many times 8 may be taken from 32; or, which is the same thing, to see how many times 8 is contained in 32. It is contained four times. *Ans.* 4 each.
>
> A boy having 32 apples was able to give 8 to each of his companions. How many companions has he?
>
> This question, though different from the other, we perceive is to be performed exactly like it. That is, it is the question to see how many times 8 is contained in 32. We take away 8 for one boy, and then 8 for another, and so on.
>
> A man having 54 cents, laid them all out for oranges at 6 cents apiece. How many did he buy?
>
> It is evident that as many times as 6 cents can be taken from 54 cents, so many oranges he can buy. *Ans.* 9 oranges.
>
> A man bought 9 oranges for 54 cents: How much did he give apiece?
>
> In this example we wish to divide the number 54 into 9 equal parts, in the same manner as in the first question we wish to divide 32 into 8 equal parts. Let us observe, that if the oranges had been only one cent apiece, nine of them would come to nine cents; if they had been 2 cents apiece, they would come to twice nine cents; if they had been 3 cents apiece, they would come to 3 times 9 cents, and so on. Hence the question is to see how many times 9 is contained in 54. *Ans.* 6 cents apiece.
>
> In all the above questions the purpose was to see how many times a small number is contained in a larger one, and they may be performed by subtraction. If we examine them again, we shall find also that the question was, in the two first, to see what number 8 must be multiplied by in order to produce 32; and in the third to see what

the number 6 must be multiplied by to produce 54; in the fourth, to see what number 9 must be multiplied by, or rather what number must be multiplied by 9, in order to produce 54.

The operation by which questions of this kind are performed is called *division*. In the last example, 54, which is the number to be divided, is called the *dividend*; 9, which is the number divided by, is called the *divisor*; and 6, which is the number of times 9 is contained in 54, is called the *quotient*.

Colburn then goes on to tell how to prove division, and following this takes up the case when the combination is not one that has occurred in the multiplication table.

At 3 cents apiece, how many pears may be bought for 57 cents? It is evident that as many pears may be bought as there are 3 cents in 57 cents. But the solution of this question does not appear so easy as the last on account of the greater number of times which the divisor is contained in the dividend. If we separate 57 into two parts it will appear more easy: 57 = 30 + 27.

We know by the table of Pythagoras that 3 is contained in 30 ten times, and in 27 nine times. Consequently it is contained in 57 nineteen times, and the answer is 19 pears.

This same method is explained for four more problems, in which he points out how the breaking up of the dividend may be determined. He then continues:

It is not always convenient to resolve the number into parts in this manner at first, but we may do it as we perform the operation.

In 126 days how many weeks?

Operation: 126 = 70 + 56. Instead of resolving it in this manner, we will write it down as follows:

Dividend	126	(7 Divisor.
	70	
	—	—
	56	10
	56	8
	—	—
		18 quotient.

I observe that 7 can not be contained 100 times in 126; I therefore call the two first figures on the left 12 tens, or 120, rejecting the 6 for the present. 7 is contained more than once and not so much as twice in 12; consequently in 12 tens it is contained more than 10 times and less than 20 times. I take 10 times 7, or 70, out of 126, and there remains 56. Then 7 is contained 8 times in 56 and 18 times in 126.

The development is continued through four more problems, the last only being abstract and having a divisor of five digits. The rule is then stated, the last thing in the section.

Short division is presented last as a "much abridged" method of performing division when the divisor is a small number.

Fractions arise in examples which require division when there is a remainder. For example, to tell "How many yards of cloth, at 6 dollars a yard, may be bought for 45 dollars" a fraction is necessary. In Sections XII to XXIV, inclusive, except Section XX, common fractions are treated in detail. (See table of contents in Appendix.) A conspicuous feature of this treatment is the departure from the usual order. It begins with a section in which fractions are manufactured by the pupil in solving such examples as "What part of 7 yards is 4 yards?" "What part of a gallon is a pint?" "What part of 5 dollars is 72 cents?" "What is the ratio of 28 to 9?"

Improper fractions are required to be changed to whole or mixed number in solving such examples as "If a family consume 1/3 of a barrel of flour in a week, how many barrels will last them four weeks? How many will last them 17 weeks?" The reverse operation is required in such as the following: "If 1/15 of a barrel of flour will serve a family one week, how many weeks will 2 4/15 barrels serve them? How many weeks will 18 7/15 serve them?" The multiplication of a fraction by an integer by multiplying the numerator, which comes in the following section, gives exercise upon reducing improper fractions to whole or mixed numbers.

In Section XVI Colburn groups together the division of a number into parts, as, "Bought 43 tons of iron for 4,171 dollars; how much was it a ton?"; and the multiplication of a whole number by a fraction, as, "At $4.20 per box what is the cost of 1/4 of a box of oranges?" These two problems require the same arithmetical operations. In this section are placed such problems as:

If 3 yards 3 qrs. of broadcloth cost $30.00, what will 7 yds. cost?

If the distance from Boston to Providence be 40 miles, how many times will a carriage wheel, the circumference of which is 15 ft. 6 in., turn round in going that distance?

What is 43/53 of a yard?

A merchant bought a quantity of tobacco for $250.00 and sold it so as to gain 3/10 of the first cost; how much did he sell it for?

If 25 men can do a piece of work in 17 days, in how many days will 38 men do it?

Three men hired a pasture for $42.00, the first put in four horses, the second, 6, and the third, 8. What ought each to pay?

In these problems are represented the rule of three, descending reduction of compound numbers, profit and loss, discount, and partnership. All of these require nothing more than the division of a whole number into parts or its multiplication by a fraction. The above types of problems are presented with no explanatory notes or headings. That they have to do with a variety of arithmetical topics Colburn is not concerned. But he is anxious that the pupil learn the kinds of situations which call for this operation.

From the standpoint of the mathematician it is interesting to note that Colburn comments upon calling the operation of this section, multiplication, by saying, "Multiplication, strictly speaking, is repeating the number a certain number of times, but by extension it is made to apply to this operation." Division of a fraction by a whole number and multiplication of a fraction by a fraction are presented in Section XVII. In the next section it is pointed out that a fraction may be multiplied by dividing the denominator.

Section XIX has to do with the addition and subtraction of fractions and the necessary reductions to a common denominator and to lower terms. The section contains 32 examples, of which 21 are practical. The drill upon finding the common divisor, least common multiple, and reducing fractions to a common denominator and lowest terms is given in Sections XXI and XXII.

Colburn's approach to reducing fractions to a common denominator is interesting and is eloquent of his general plan to have the pupil see what a process is for before he is asked to perform it.

> We observed a remarkable circumstance in the last article, viz, that 1/2=4/8 and 3/7=12/28. This will be found very important in what follows.
>
> A man having a cask of wine, sold 1/2 of it at one time and 1/3 of it at another, how much had he left?
>
> 1/2 and 1/3 can not be added together, because the parts are of different values. Their sum must be more than 2/3 and less than 2/2 or 1. If we have dollars and crowns to add together, we reduce them both to pence. Let us see if these fractions can not be reduced both to the same denomination. Now 1/2=2/4=3/6=4/8, etc.

The "remarkable circumstance" had come about from the two ways of multiplying a fraction. Multiplying a fraction by dividing by its denominator gave the result in lower terms than by multiplying the numerator of the fraction.

Sections XXIII and XXIV are devoted to the division of a whole number, or a fraction, by a fraction. After a rather lengthy development this generalization is reached: "Multiply the dividend by the denominator of the divisor, and divide the product by the numerator."

In the next four sections decimal fractions are presented. Their notation is explained as being simply an extension of the decimal system in which a figure has a place value and the topic is treated in Colburn's inductive manner. In general it appears that he believes operations with decimal fractions are similar to operations with whole numbers, and this is the idea he wishes the pupil to get. The only serious difficulty the pupil is going to have, as he sees it, is in division, and he develops this in detail.

The last section is concerning circulating decimals, a topic we did not find in the texts of the previous period. A circulating decimal is one such as arises when one attempts to reduce a common fraction such as 1/7 to an equivalent decimal form. One will get a never ending sequence of figures, but in this sequence certain series of figures will be repeated. After Colburn shows the occasion for circulating decimals, he explains how one may find the equivalent common fraction when they have given a circulating decimal. Except for a list of miscellaneous examples, the text closes with a brief presentation of the proof of multiplication and division by casting out nines.

Definitions and information given when needed.—We have already pointed out this feature in several instances. It is one of the chief characteristics of both of Colburn's texts. Colburn's treatment of percentage, interest, etc., is perhaps most typical of this feature and of his attitude toward the applications of arithmetic. On page 21 of the Sequel, in the section on multiplication, this paragraph is given immediately preceding the first problem on interest:

Interest is a reward allowed by a debtor to a creditor for the use of money. It is reckoned by the hundred, hence the rate is called so much *per cent* or *per centum*. Per centum is Latin, signifying by the hundred. 6 per cent signifies 6 dollars on a hundred dollars, 6 cents on a hundred cents, £6 on £100, etc., so 5 per cent signifies 5 dollars on 100 dollars, etc. Insurance, commission, and premiums of every kind are reckoned in this way. Discount is so much per cent to be taken out of the principal.

Colburn evidently considers this sufficient explanation for such problems as the following, for he gives nothing additional either here or in Part II:

What is the interest of $43.00 for 1 year at 6 per cent?

What is the interest of $247.00 for 3 years at 7 per cent?

Imported some books from England, for which I paid $150.00 there. The duties in Boston were 15 per cent, the freight $5.00. What did the books cost me?

A merchant bought a quantity of goods for 243 dollars, and sold them so as to gain 15 per cent; how much did he gain, and how much did he sell them for?

The next mention of percentage is on page 83. This problem is given:

A merchant sold a quantity of goods for $273.00, by which he gained 10 per cent on the first cost. How much did they cost?

Following the problem is this note:

10 per cent is 10 dollars on a 100 dollars, that is 10/100. 10 per cent of the first cost therefore is 10/100 of the first cost. Consequently $273.00 must be 110/100 of the first cost.

A little farther on in the list we find the following problems and notes:

A merchant sold a quantity of goods for $983.00, by which he lost 12 per cent. How much did the goods cost and how much did he lose?

Note.—If he lost 12 per cent, that is 12/100, he must have sold for 88/100 of what it cost him.

A merchant sold a quantity of goods for $87.00 more than he gave for them, by which he gained 13 per cent of the first cost. How much did the goods cost him, and how much did he sell them for?

Note.—Since 13 per cent is 13/100, $87.00 must be 13/100 of the first cost.

A man having put a sum of money at interest at 6 per cent, at the end of 1 year received 13 dollars for interest. What was the principal?

Note.—Since 6 per cent is 6/100 of the whole, 13 dollars must be 6/100 of the principal.

A man put a sum of money at interest for 1 year at 6 per cent, and at the end of the year he received for the principal and interest 237 dollars. What was the principal?

Note.—Since 6 per cent is 6/100, if this be added to the principal it will make 106/100, therefore $237 must be 106/100 of the principal. When interest is added to the principal, the whole is called the *amount*.

What sum of money put at interest at 6 per cent will gain $53 in 2 years?

Note.—6 per cent for 1 year will be 12 per cent for 2 years, 3 per cent for 6 months, 1 per cent for 2 months, etc.

Suppose I owe a man $287, to be paid in one year without interest, and I wish to pay it now; how much ought I to pay him when the usual rate is 6 per cent?

Note.—It is evident that I ought to pay him such a sum as put at interest for 1 year will amount to $287. The question therefore is like those above. This is sometimes called *discount*.

Later in the sections on decimal fractions special methods for interest are given in the same way, i. e., by means of a note following a problem which calls for a special method.

Chapter VI.

COLBURN ON THE TEACHING OF ARITHMETIC.

In the preface to his texts and in his address on the teaching of arithmetic[1] Colburn has given a good presentation of his method of teaching arithmetic. These accounts are supplemented by the texts. In this chapter we shall present the most significant features of his method.

The pupil is introduced to a topic by means of practical problems.—Colburn's introduction of the pupil to arithmetic is in striking contrast to that in the texts used prior to 1821. (See p. —.) For example, the first page of the First Lessons is as follows:

1. How many thumbs have you on your right hand? How many on your left? How many on both together?
2. How many hands have you?
3. If you have two nuts in one hand and one in the other, how many have you in both?
4. How many fingers have you on one hand?
5. If you count the thumb with the fingers, how many will it make?
6. If you shut your thumb and one finger and leave the rest open, how many will be open?
7. If you have two cents in one hand, and two in the other, how many have you in both?
8. James has two apples, and William has three; if James gives his apples to William, how many will William have?
9. If you count all the fingers on one hand, and two on the other, how many will there be?
10. George has three cents, and Joseph has four; how many have they both together?

These problems are followed by 22 of similar nature, and these in turn are followed by 163 drill questions on the combinations. This plan is continued through the book.

Use of symbols delayed.—One phase of the organization of the subject matter is Colburn's treatment of the symbols of notation which seems to exemplify one of his fundamental notions of arithmetic. For example, he wishes the pupil to learn that two objects and one object make a total of three objects; that five plums and four plums are nine plums, and *not* that the symbols $2+1$ equal the

[1] This address was delivered before the American Institute of Instruction in Boston, August, 1830. It was published in the proceedings of that society and was reprinted in the Elementary School Teacher for June, 1912.

symbol 3, or the symbols 5+4 equal the symbol 9. As a means to this end, in the First Lessons, the characters 1, 2, 3, etc., are not given until page 50, and the system of notation and numeration is not given beyond 10 until page 69. Before these symbols and the system of notation and numeration are given, the pupil has learned the four fundamental operations for integers. The symbols are introduced by saying, "Instead of writing the names of numbers, it is usual to express them by particular characters called figures." Thus before the pupil is asked to learn number symbols, he doubtless has felt the need for them.

In giving his reason for these two features Colburn says, referring to the contemporary practice:

> The following are some of the principal difficulties which a child has to encounter in learning arithmetic, in the usual way, and which are seldom overcome. First, the examples are so large that the pupil can form no conception of the numbers themselves; therefore it is impossible for him to comprehend the reasoning upon them. Secondly, the first examples are usually abstract numbers. This increases the difficulty very much, for even if the numbers were so small that the pupil could comprehend them, he would discover but very little connection between them and practical examples. Abstract numbers, and the operations upon them, must be learned from practical examples; there is no such thing as deriving practical examples from those which are abstract, unless the abstract have been first derived from those which are practical. Thirdly, the numbers are expressed by figures, which, if they were used only as a contracted way of writing numbers, would be much more difficult to be understood at first than the numbers written at length in words. But they are not used merely as words, they require operations peculiar to themselves. They are, in fact, a new language, which the pupil has to learn. The pupil, therefore, when he commences arithmetic is presented with a set of abstract numbers, written with figures, and so large that he has not the least conception of them even when expressed in words. From these he is expected to learn what the figures signify, and what is meant by addition, substraction, multiplication, and division; and at the same time how to perform these operations with figures. The consequence is, that he learns only one of all these things, and that is, how to perform these operations on figures. He can perhaps translate the figures into words, but this is useless since he does not understand the words themselves. Of the effect produced by the four fundamental operations he has not the least conception.
>
> After the abstract examples a few practical examples are usually given, but these again are so large that the pupil can not reason upon them, and consequently he could not tell whether he must add, substract, multiply, or divide, even if he had an adequate idea of what these operations are.
>
> The common method, therefore, entirely reverses the natural process; for the pupil is expected to learn general principles before he has obtained the particular ideas of which they are composed.[1]

Oral instruction.—Just as the most conspicuous feature of the method of teaching arithmetic during the ciphering-book period was the absence of a textbook in the hands of the pupil, and the consequent exclusively written arithmetic, so the most conspicuous feature

[1] First Lessons, preface.

of Colburn's method is *oral instruction*, or the solving of exercises in the mind. Colburn does not provide for written computations in the First Lessons. In fact, as we have mentioned, he does not introduce the number symbols at all in the first third of the book. The quantities of the problems throughout the book are small enough to bring the numbers within the comprehension of the pupil and also so small that he may solve the problems mentally. It is therefore probable that pupils solved the problems of the First Lessons without recourse to written calculations. When there were no "sums" to be done on paper or slate and submitted to the teacher for inspection, it became necessary for the teacher to hear the pupils give an oral solution of the problem. Thus, at least in the case of the younger pupils, instruction in arithmetic was largely oral after the appearance of the First Lessons. The Sequel was a "written arithmetic," but in it close connection is made between "operations performed in the mind" and the "application of figures to these operations."

From concrete to abstract.—Colburn invariably introduces a topic or a new combination by a "practical question." In the case of a new combination the "practical question" is followed by the same combination in abstract form. For example, the multiplication of an integer by a fraction is begun as follows:

If a yard of cloth costs 3 dollars, what will 1 half of a yard cost?
What is 1 half of 3?
If a barrel of beer costs 5 dollars, what will 1 half of a barrel cost?
What is 1 half of 5?

In the preface to the First Lessons the necessity of this order in teaching children is emphasized:

The idea of number is first acquired by observing sensible objects. Having observed that this quality is common to all things with which we are acquainted, we obtain an abstract idea of number. We first make calculations about sensible objects; and we soon observe that the same calculations will apply to things very dissimilar; and finally, that they may be made without reference to any particular things. Hence from particulars we establish general principles, which serve as the basis of our reasonings and enable us to proceed step by step, from the most simple to the most complex operations. It appears, therefore, that mathematical reasoning proceeds as much upon the principle of analytic induction as that of any other science.

Examples of any kind upon abstract numbers are of very little use until the learner has discovered the principle from practical examples. They are more difficult in themselves, for the learner does not see their use, and therefore does not so readily understand the question. But questions of a practical kind, if judiciously chosen, show at once what the combination is, and what is to be effected by it. Hence the pupil will much more readily discover the means by which the result is to be obtained. The mind is also greatly assisted in the operations by reference to sensible objects. When the pupil learns a new combination by means of abstract examples, it very seldom happens that he understands practical examples more easily for it, because he does not discover the connection until he has performed several practical examples and begins to generalize them.

And it is not too bold an assertion to say that no man ever actually learned mathematics in any other method than by analytic induction; that is, by learning the principles by the examples he performs, and not by learning principles first, and then discovering by them how the examples are to be performed.

The full significance of this feature of Colburn's method appears only when we compare it with the practice of his time. It marks, as do other features of his work, an absolute break with the past. The principle is fundamental with him, and its effect is clearly evident throughout both texts as well as in his method of teaching.

Objective method.—In the First Lessons the pupil is not told the "combinations," but he is expected to discover them by using objective materials, the Pestalozzian tables, or beans, peas, etc., in performing the operations which the "practical questions" called for. The advantage of asking the child to think in terms of concrete objects is mentioned in the above quotation. It should be noted that Colburn recommends the use of objective material only when a pupil has need of it. It is not his purpose to introduce objective material for the purpose of amusing pupils, and he intends that they shall transcend the use of it. The objective method, next to the oral instruction, is the most conspicuous feature of Colburn's method of teaching.

Assisting the pupil.—It has already been indicated that Colburn had a definite and accurate conception of the working of the human mind. He also knew the appropriate manner in which to assist this working. This he discusses in the preface to the Sequel:

When the pupil is to learn the use of figures for the first time, it is best to explain to him the nature of them to about three or four places, and then require him to write some numbers. Then give him some of the first examples without telling him what to do. He will discover what is to be done, and invent a way to do it. Let him perform several in his own way, and then suggest some method a little different from his, and nearer the common method. If he readily comprehends it, he will be pleased with it, and adopt it. If he does not, his mind is not yet prepared for it, and should be allowed to continue his own way longer, and then it should be suggested again. After he is familiar with that, suggest another method somewhat nearer the common method, and so on, until he learns the best method. Never urge him to adopt any method until he understands it and is pleased with it. In some of the articles it may perhaps be necessary for young pupils to perform more examples than are given in the book.

One general maxim to be observed with pupils of every age is never to tell them directly how to perform any example. If a pupil is unable to perform an example, it is generally because he does not fully comprehend the object of it. The object should be explained, and some questions asked which will have a tendency to recall the principles necessary. If this does not succeed, his mind is not prepared for it, and he must be required to examine it more by himself, and to review some of the principles which it involves. It is useless for him to perform it before his mind is prepared for it. After he has been told, he is satisfied, and will not be willing to examine the principle, and he will be no better prepared for another case of the same kind than he was before. When the pupil knows that he is not to be told, he learns to depend upon himself; and when he once contracts the habit of understanding what he does, he will not easily be prevailed on to do anything which he does not understand.

Also in his address he speaks at length upon how the teacher should assist the pupil:

If the learner meets with a difficulty, the teacher, instead of telling him directly how to go on, should examine him and endeavor to discover in what the difficulty consists; and then, if possible, remove it. Perhaps he does not fully understand the question. Then it should be explained to him. Perhaps it depends upon some former principle which he has learned, but does not readily call to mind. Then he should be put in mind of it. Perhaps it is a little too difficult. Then it should be simplified. This may be done by substituting smaller numbers, or by separating it into parts and making a distinct question of each of the parts. Suppose the question were this: If 8 men can do a piece of work in 12 days, how long would it take 15 men to do it? It might be simplified by putting in smaller numbers, thus: If 2 men can do a piece of work in 3 days, how long would it take 5 men to do it? If this should still be found too difficult, say, If 2 men can do a piece of work in 3 days, how long will it take 1 man to do it? This being answered, say, If 1 man will do it in 6 days, how long will it take 3 men to do it? In what time would 4 men do it? In what time would 5 men do it? By degrees, in some such way as this, lead him to the original question. Some mode of this kind should always be practiced; and by no means should the learner be told directly how to do it, for then the question is lost to him. For when the question is thus solved for him, he is perfectly satisfied with it, and he will give himself no further trouble about the mode in which it is done.

All illustrations should be given by practical examples, having reference to sensible objects. Most people use the reverse of this principle and think to simplify practical examples by means of abstract ones. For instance, if you propose to a child this simple question: George had 5 cents, and his father gave him 3 more, how many had he then? I have found that most persons think to simplify such practical examples by putting them into an abstract form and saying, How many are 5 and 3. But this question is already in the simplest form that it can be. The only way that it can be made easier is to put it into smaller numbers. If the child can count, this will hardly be necessary. No explanation more simple than the question itself can be given, and none is required. The reference to sensible objects, and to the action of giving, assists the mind of the child in thinking of it, and suggests immediately what operation he must perform; and he sets himself to calculate it. He has not yet learned what the sum of those two numbers is. He is therefore obliged to calculate it in order to answer the question, and he will require some little time to do it. Most persons, when such a question is proposed, do not observe the process going on in the child's mind; but because he does not answer immediately, they think that he does not understand it, and they begin to assist him, as they suppose, and say, How many are 5 and 3? Can not you tell how many 5 and 3 are? Now this latter question is very much more difficult for the child than the original one. Besides, the child would not probably perceive any connection between them. He can very easily understand, and the question itself suggests it to him better than any explanation, that the 5 cents and 3 cents are to be counted together; but he does not easily perceive what the abstract numbers 5 and 3 have to do with it. This is a process of generalization which takes children some time to learn.

In all cases, especially in the early stages, it will be perplexing and rather injurious to refer the learner from a practical to an abstract question for the purpose of explanation. And it is still worse to tell him the result, and not make him find it himself. If the question is sufficiently simple, he will solve it. And he should be allowed time to do it and not be perplexed with questions or interruptions until he has done it.

But if he does not solve the question, it will be because he does not fully comprehend it. And if he can not be made to comprehend it, the question should be varied, either by varying the numbers, or the objects, or both, until a question is made that he can answer. One being found that he can answer, another should be made a little varied and then another, and so on till he is brought back to the one first proposed. It will be better that the question remain unanswered than that the child be told the answer, or assisted in the operation any further than may be necessary to make him fully understand the question.

It is clear that Colburn understood that a dfficulty initiates reflective thought. The pupil is at first to meet a difficulty, feel a need, have a problem. This is the first step. Second, the pupil is to make his own hypothesis; the teacher is to keep hands off. Unless the problem is one for which the pupil is not prepared, he will "invent" a way to solve the problem. It may be a crude one, but nevertheless a method which will control the value. The thought process involved here is that of making hypotheses and verifying them. The instructor is in the background. Colburn would have his function to be that of explaining to the pupil the meaning of the problem and its demands, and to see that the pupil was finally made acquainted with the best method of solving the problem.

Inductive instruction.—In the titles of both of his arithmetics, Colburn explicitly states that the method of presentation is inductive rather than deductive. His inductive development is not formal and mechanical, but here as elsewhere he has grasped the manner of the working of the human mind. The complete texts must be studied to appreciate fully the quality of his inductive development of a topic, but the development of division in the Sequel will give an idea of the charm of Colburn's inductive treatment of a topic. (See p. 74.)

This is as near real induction as it is possible to get in a textbook. The pupil is given problems which he can understand and appreciate; the first he may solve in a crude fashion, more difficult problems force him to make hypotheses, and the rule is delayed so that the pupil has had an opportunity to test his hypothesis empirically. As a consequence, the pupil probably has discovered the appropriate rule before he reaches the statement of it in the text.

Class instruction.—During the ciphering-book period, the instruction of necessity was individual. Before 1821 the need was being keenly felt for a more expeditious manner of teaching arithmetic. The attendance was increasing very rapidly, and arithmetic was beginning to be taught quite generally to all pupils. This condition made it necessary to instruct the pupils in groups. Colburn not only advocated class instruction, but gives suggestions as to the technique.

It is chiefly at recitation that one scholar can compare himself with another; consequently they furnish the most effectual means of promoting emulation. They are an excellent exercise for the scholar, for forming the habit of expressing his ideas properly and readily. The scholar will be likely to learn his lesson more thoroughly when he knows he shall be called upon to explain it. They give him an opportunity to discover whether he understands his subject fully or not, which he will not always be sure of, until he is called upon to give an account of it. Recitations in arithmetic, when properly conducted, produce a habit of quick and ready reckoning on the spur of the occasion, which can be produced in no other way except in the business of life, and then only when the business is of a kind to require constant practice. They are therefore a great help in preparing scholars for business.

Directions concerning recitations must be general. Each teacher must manage the detail of them in his own way.

In the first place, the scholar should be thoroughly prepared before he attempts to recite. No lessons should be received by the teacher that are not well learned. If this is not insisted on, the scholar will soon become careless and inattentive.

It is best that the recitations, both in intellectual and written arithmetic, should be in classes when practicable. It is best that they should be without the book, and that the scholar should perform the examples from hearing them read by the teacher. Questions that are put out to be solved at the recitation should be solved at the recitation, and not answered from memory. The scholars should frequently be required to explain fully and clearly the steps by which they solve a question and the reasons for them. Recitations should be conducted briskly and not suffered to lag and become dull. The attention of every scholar should be kept upon the subject, if possible, so that all shall hear every thing that is said. For this it is necessary that the questions pass around quickly, and that no scholar be allowed a longer time to think than is absolutely necessary. If the lesson is prepared as it should be, it will not take the scholar long to give his answer. It is not well to ask one scholar too many questions at a time, for by that there is danger of losing the attention of the rest. It is a good plan, when practicable, so to manage the recitations that every scholar shall endeavor to solve each question that is proposed for solution at the time of the recitation. This may be done by proposing the question without letting it be known who is to answer it until all have had time to solve it, and then calling upon someone for the answer. No further time should be allowed for the solution; but if the scholar, so called on is not ready, the question should be immediately put to another in the same manner.[1]

He also shows a trace of the monitorial system when he says:

It will often be well to let the elder pupils hear the younger. This will be a useful exercise for them, and an assistance to the instructor.

Teaching pupils to study.—Colburn recognized the value of teaching pupils how to study. He says:

There is one more point which I shall urge, and it is one which I consider the most important of all. It is to make the scholars study. I can give no directions how to do it. Each teacher must do it in his own way, if he does it at all. He who succeeds in making his scholars study will succeed in making them learn, whether he does it by punishing, or hiring, or persuading, or by exciting emulation, or by making the studies so interesting that they do it for the love of it. It is useless for me to say which will produce the best effects upon the scholars; each of you may judge of that for yourselves. But this I say, that the one who makes his scholars study will make them learn; and he who does not will not make them learn much or well. There

[1] Address, "The Teaching of Arithmetic."

never has been found a royal road to learning of any kind, and I presume there never will be. Or if there should be, I may venture to say that learning so obtained will not be worth the having. It is a law of our nature, and a wise one too, that nothing truly valuable can be obtained without labor.[1]

In another place he suggests some necessary conditions:

This subject also suggests a hint with regard to making books, and especially those for children. The author should endeavor to instruct by furnishing the learner with occasions for thinking and exercising his own reasoning powers, and he should not endeavor to think and reason for him. It is often very well that there should be a regular course of reasoning in the book on the subject taught; but the learner ought not to be compelled to pursue it, if it can possibly be avoided, until he has examined the subject and come to a conclusion in his own way. Then it is well for him to follow the reasoning of others, and see how they think of it.[1]

Motivation.—Although Colburn recognized that there were several ways for making arithmetic interesting, he selected the problems which especially appeal to children and caused them to feel a need for a process or definition before it is given. The types of problems are well illustrated by those already given. A feeling of need for the process is created by introducing each topic by problems. The very plan of dividing the texts into two parts, and thus separating the problems from the development of the principles, operates to create motive for the study of the principles. Even in the development of the principles, the rules are not stated until after the explanation of the operation which is itself based upon a problem. Whatever drill seems necessary is not given until after a considerable number of practical problems have been solved by the pupils.

But even these devices do not represent all that Colburn has done to motivate the arithmetic work. His style of writing and his ability to see things from the child's point of view assist materially in this respect, and the way he guides the learner in the development of the principles adds a touch of genius to the whole work. The following is from the Sequel, p. 193:

A boy wishes to divide $\frac{3}{4}$ of an orange equally between two other boys; how much must he give them apiece?

If he had three oranges to divide, he might give them one apiece and then divide the other into two equal parts, and give one part to each, and each would have $1\frac{1}{2}$ orange. Or he might cut them all into two equal parts each, which would make six parts, and give three parts to each, that is, $\frac{3}{2}=1\frac{1}{2}$, as before. But according to the question, he has $\frac{3}{4}$ or 3 pieces, consequently he may give 1 piece to each, and then cut the other into two equal parts, and give 1 part to each, then each will have $\frac{1}{4}$ and $\frac{1}{2}$ of $\frac{1}{4}$. But if a thing be cut into four equal parts and then each part into two equal parts, the whole will be cut into 8 equal parts or eighths; consequently $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$. Each will have $\frac{1}{4}$ and $\frac{1}{8}$ of an orange. Or he may cut each of the three parts into two equal parts, and give $\frac{1}{2}$ of each part to each boy, and then each will have 3 parts, that is $\frac{3}{8}$. Therefore $\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{3}{8}$. *Ans.* $\frac{3}{8}$.

[1] Address, "The Teaching of Arithmetic."

Two more problems are similarly explained, though somewhat more briefly. He then draws a conclusion as follows:

In the last three problems the division is performed by multiplying the denominator. In general, if the denominator of a fraction be multiplied by 2, the unit will be divided into twice as many parts, consequently the parts will be only one-half as large as before, and the same number of the small parts be taken, as was taken of the large, the value of the fraction will be one-half as much. If the denominator be multiplied by three, each part will be divided into three parts, and the same number of parts be taken, the fraction will be one-third of the value of the first. Finally, if the denominator be multiplied by any number, the parts will be so many times smaller. Therefore, to divide a fraction, if the numerator can not be divided exactly by the divisor, multiply the denominator by the divisor.

PART III. THE INFLUENCE OF WARREN COLBURN IN DIRECTING THE DEVELOPMENT OF ARITHMETIC AS A SCHOOL SUBJECT. ACTIVE PERIOD, 1821-1857; STATIC PERIOD, 1857-1892.

Chapter VII.

ARITHMETIC AS A MENTAL DISCIPLINE

During the first half of the nineteenth century the growth of cities, the rise of manufacturing, the invention of machines, new modes of travel and transportation, and other factors combined to produce a demand for a higher degree of education than had been necessary when life was more simple. At the same time, the home began to contribute less to the child's education. As a consequence there came to be a new concept of the purpose and scope of the education provided by the schools and an awakened interest in public schools. This movement which has been known as "the common-school revival" was most prominent between 1835 and 1850. The interest in the work of Pestalozzi, which we have noted in Chapter IV, the production of texts by American authors,[1] and the extension of the public-school system to include primary schools and high schools were phases of the larger movement.

The production of arithmetic texts by American authors, the modification of the content of arithmetic, the extension of the instruction in the subject, and the attempts to provide texts for young children were elements in the general development of arithmetic as a school subject in the United States. This movement had been growing since the close of the Revolutionary War, and the adoption of a Federal money was a phase of the "great awakening." In the three preceding chapters we have told of Colburn's contribution. It is the problem of this chapter and the two following to show in what ways and to what extent Warren Colburn augmented and directed this development.

The limits of the period.—The importance of Colburn's First Lessons justifies the selection of 1821 as marking the beginning of this period in the development of arithmetic as a school subject. Following this date there was a period of very rapid development. New types of texts appeared. Some of these were revised frequently to keep pace with the growing ideas of the time. But, beginning about 1860, these revisions ceased; and after this date it is seldom that we find a new type of text which attained any importance.

[1] Arithmetic texts by American authors have been mentioned on page 14.

There was no great event, such as the appearance of Colburn's First Lessons, to mark the close of this period. At times from 1821 to 1892 innovations were attempted, some acquiring a considerable following. However, after about 1860, there was no essential change in the aim or content nor modification in the method of teaching which was not local or merely temporary until well toward the close of the century. Then new types of texts became popular and replaced those which had been used for over a quarter of a century. Also radical changes in the method of teaching were urged. Several events indicate that the date of this transition was about 1890. We have chosen 1892, the date of the Report of the Committee of Ten. Although this report dealt only incidentally with arithmetic, it was the official declaration of the teachers of the United States and marked the alignment of a number of our greatest educators on the side of arithmetical reform.

The date marking the end of the process of formalization and the beginning of a stationary period is likewise difficult to determine with exactness. We have chosen 1857, the date of the last of a series of revisions of Ray's arithmetics. Just prior to this date, revisions of Ray's arithmetics were frequent, but in 1857 a form was attained which was not altered until 1877 and only slightly then. Other texts and events do not, in general, specify the date 1857, but they agree in indicating the beginning of a relatively static period about 1860. In view of the popularity and the widespread and continued use of Ray's arithmetics it is appropriate that we select the date marking their maturity.

Mental arithmetic.[1]—The arithmetic of the preceding period was confined to calculations with written symbols. There were no examples or problems in which the quantities were small to be solved without the use of pencil or pen. In fact, the subject was frequently spoken of as "ciphering." Colburn intended that the problems of his First Lessons should be solved without the aid of written symbols, and he constructed the book in such a way that this was made necessary unless the teacher supplemented the text by instructions in "written arithmetic."

After 1821 the more popular arithmetics were issued in the form of a series. Usually one book of such a series was devoted to mental arithmetic. A few authors united the two types of arithmetic in the same text. Mental arithmetic was universally taught, frequently in a course paralleling the one in written arithmetic.

[1] The term "mental arithmetic" became quite generally used to designate that arithmetic which did not involve computations with written symbols. Colburn and some other authors used the term, "intellectual" instead of "mental," and still others called this type of arithmetic, "oral." The use of the term, "mental arithmetic," has been criticized on the ground that arithmetic which involves calculations with written symbols is just as truly mental as that which does not, but the term has been and is still so generally used that its use here is justified and will serve to avoid confusion.

Texts for young children.—The texts of the preceding period were not suitable for young children. Thus when arithmetic was taught to them no text was used in the hands of the pupils. It was only a few years prior to 1821 that there was an attempt to provide a text for young children.[1] But soon after 1821 many primary books appeared and a series of arithmetics was not complete unless it contained a text specifically intended for young children. There were texts prepared to precede Colburn's First Lessons, which Colburn claimed was simple enough for children 5 or 6 years of age.

Most of the primary texts embodied the use of objects. In many of them there were pictures in which the pupil was to count the number of objects. In some texts examples were represented graphically by means of marks, dots, etc., or by actual pictures of the objects mentioned in the exercise.

Arithmetic as a mental discipline.—Throughout the preceding period, as we have shown, arithmetic was taught because of its practical value in certain trades and commerce. A disciplinary function of arithmetic was emphasized by Pestalozzi, who believed that it was to be attained by drill upon a set of abstract exercises which were to be solved by the use of his tables or other sensible objects. Colburn recognized mental discipline as one of the important functions to be realized from the study of arithmetic. The recognition of the disciplinary function, particularly as attached to mental arithmetic, grew after the appearance of Colburn's First Lessons until it overshadowed the other functions. Davies says in the preface to his *School Arithmetic*, 1855: "In the preparation of this work, two objects have been kept constantly in view: First, to make it educational; second, to make it practical." The educational value which Davies has in mind here is mental discipline. Joseph Ray says in the preface of his *Intellectual Arithmetic*, one-thousandth edition, 1860:

> By its (mental arithmetic) study, learners are taught to reason, to analyze, to think for themselves, while it imparts confidence in their reasoning powers and strengthens the mental faculties.

Davies puts it somewhat more forcibly in his Intellectual Arithmetic:

> It is the object of this book to train and develop the mind by means of the science of numbers. Numbers are the instruments here employed to strengthen the memory, to cultivate the faculty of abstraction, and to sharpen and develop the reasoning powers.

In the *New Normal Mental Arithmetic*, by Edward Brooks, 1873, the author says:

> The science of arithmetic, until somewhat recently, was much less useful as an educational agency than it should have been. Consisting mainly of rules and methods

[1] See p. 39.

of operations, without presenting the reasons for them, it failed to give that high degree of mental discipline which, when properly taught, it is so well calculated to afford. But a great change has been wrought in this respect; a new area has dawned upon the science of numbers; a "royal road" to mathematics has been discovered, so graded and strewn with the flowers of reason and philosophy that the youthful learner can follow it with interest and pleasure; and one of the most influential agents in this work has been the system of mental arithmetic.

The importance of this change can hardly be overestimated. The study of mental arithmetic, introduced by Warren Colburn, to whom teachers and pupils owe a debt of gratitude which can never be paid, affords the finest mental discipline of any study in the public schools. When properly taught, it gives quickness of perception, keenness of insight, toughness of mental fiber, and an intellectual power and grasp that can be acquired by no other elementary branch of study. An old writer on arithmetic quaintly called his work "The Whetstone of Wit." Mental arithmetic is, in my opinion, truly a *whetstone of wit*. It is a mental grindstone; it sharpens the mind and gives it the power of concentration and penetration. To omit a thorough course of mental arithmetic in the common school is to deprive the pupil of one of the principal sources of mental power.

Arithmetic as a science.—Since the time of the Greek philosophers arithmetic has been conceived of both as an art and as a science, or as some authors put it, as practical arithmetic and theoretical arithmetic. The writers of the texts which were used during the ciphering-book period usually recognized both of these aspects of arithmetic, but they seem to have done so mainly for traditional reasons. In the schools arithmetic was an art. But in this period a number of texts became colored with a philosophic point of view. The theoretical part of arithmetic was given more emphasis. The principles were more carefully formulated, and special attention was given to their interrelation and organization into a logical system. Greenleaf in the *National Arithmetic* (first published 1835, revised 1847, 1857) gives elaborate lists of definitions, axioms, and principles, and a chapter on properties of numbers. By some writers the "science of numbers" is used synonymously with arithmetic.[1]

Arithmetic, the important school subject.—By reason of more simple texts and by reason of the emphasis upon the disciplinary function of arithmetic, its relative importance as a school subject grew during this period. It became the custom for pupils to receive instruction in arithmetic when they began to attend school, which in some cases was before their fourth birthday.[2]

Frequently, mental arithmetic was recognized as a separate subject, and two periods a day were given to arithmetic in several of the grades, in some schools from the third or fouth grade to the eighth, inclusive. William B. Fowle said in 1866:

> Arithmetic is the all-absorbing study in the public schools of Massachusetts, and, probably, in those of every other State. As far as my observation goes, it occupies more of the time of our children than all other branches united.[3]

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[1] See the citations from Davies and Brooks, quoted above.

[2] Clifton Johnson: Old Time Schools and School Books, p. 37.

[3] The Teacher's Institute; or Familiar Hints to Young Teachers, p. 45.

Another writer said:

Having such prominence, the subject came to be taken as the basis of gradation and of promoting pupils.[1]

It is difficult for the teachers of to-day to realize that arithmetic has not always been one of the fundamentals of the school curriculum. There is the general impression that the curriculum consisted of the three R's until it was enriched by the addition of the more modern subjects. Hence we fail to appreciate that it was not until the second quarter of the nineteenth century that arithmetic was accorded its place in our schools as one of the traditional educational trinity.

Inductive method.—The complete title to Colburn's First Lessons[2] contained the phrase, "on the inductive method of instruction," and this method was a conspicuous feature of his texts. During the active period from 1821 to 1857, authors frequently included some reference to the inductive method in the title of their texts. In the construction of their texts many followed closely Colburn's plan. Some authors adhered to the deductive plan, and after 1857 the texts, even those which had previously embodied the "inductive method," were generally organized deductively.

Skill and thoroughness.—Increasing emphasis was placed upon skill in performing the operations of arithmetic. This is testified to by the increased space given to drill exercises and the publication of "Lightning Calculators," which were numerous in the last half of the century. In the preface to the *New Intermediate Arithmetic*, Felter says:

This book is designed to make the pupil quick and accurate in calculation, and to give him a knowledge of those principles and processes of arithmetic which are needed in the ordinary transactions of life, together with skill in their application.

To accomplish this, the drill card exercises are arranged to furnish any desired amount of practice in computation; while the processes and analyses leading directly to the rule, together with the number, gradation, and character of the practical examples, give the knowledge of necessary principles and skill in their use.

Felter says in the preface to *An Introduction to Arithmetical Analysis:* "The importance of being thorough in the elements of arithmetic can not be too often impressed upon the teacher."

In brief these are the significant features of arithmetic as a school subject in this period. In each of them there are evidences of Colburn's influence. In the next chapter the important texts of the period are described, and in them we shall see more clearly the influence of Colburn upon the arithmetic of this period.

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[1] J. M. Greenwood: Principles of Education Practically Applied, 1887, p. 154.

[2] Edition of 1826.

Chapter VIII.

FORMALIZED PESTALOZZIAN ARITHMETICS.

In this period, arithmetic was usually presented in a series of three books for the common schools and a higher arithmetic which was primarily for academies and colleges. While there was no absolute uniformity in the planes of division of arithmetic in the common schools, yet in general there was, first, a primary arithmetic which covered the work pursued in the primary school, which varied from two and one-half years to four years; second, an intellectual, or mental arithmetic; and third, a text often designated as "practical," or "common school," and which was a complete text in itself. In case a text became at all popular, it was provided with a key for the use of the teacher. The originally distinct line of cleavage between mental arithmetic and written arithmetic became less and less distinct by reason of combining the two, which was especially popular in the latter portion of this period.

Two classes of texts of this period will be described; first, those which were widely used and hence exemplify the practice of the times, and second, other important texts. This second class contains texts which were not used as extensively as the first class, but which indicate the course of development. The extensiveness of the use of a text has been determined by considering the number of editions through which the text passed and the length of time it was before the public.

For description the texts have been grouped under authors. The order is determined by the date of the first text by an author.

ARITHMETICS WHICH WERE WIDELY USED.

Warren Colburn's First Lessons, 1821, which we have described, was the most extensively used mental arithmetic during the active portion of this period and must be counted among important texts of the entire period.

Frederick Emerson (1788–1857), who wrote the *North American Arithmetics*, was for a number of years a teacher in the Boston public schools, later principal in the department of arithmetic at Boylston School, and finally superintendent of schools. The *Part First* appeared in 1827, *Part Second* in 1832, and *Part Third* in 1834. Part First is distinctly an elementary book, and the author states, "The slate and pencil are not required in the performance of the lessons contained in Part First." The first part of the Part Second consists of oral arithmetic, and the second of written arithmetic. Part Third is designed for advanced scholars, and as such is a scholarly presentation of the subject from a mature point of view.

As soon as the series was complete, it displaced Colburn's texts in the Boston schools, and the *North American Arithmetics, Part First,* was an alternative text as late as 1866–67. The series had been used in Chicago preceding 1866. In an edition of Part First, it is stated that it has been adopted in Boston, Salem, Portland, Providence, New York, Philadelphia, and Louisville. I have examined copies of Part Second dated 1832, 1839, 1848, 1854, and of Part Third dated 1834, 1844, 1850. Part Third appeared in two forms, both copyrighted in 1834. One of these is announced as revised and enlarged. The enlargement is a list of questions for examination. Otherwise the series does not appear to have been revised.

Charles Davies (1798–1876) graduated from the Military Academy at West Point in 1815. He was professor of mathematics and natural philosophy in that institution from 1823 to 1837, and professor of mathematics in Trinity College, Hartford, 1839 to 1841. Later he taught mathematics in the normal school at Albany, N. Y., and was professor of higher mathematics in Columbia College, New York City from 1857 to 1867, when he was made emeritus professor. Davies's primary arithmetic was published under the title of *First Lessons in Arithmetic* in 1840. There is also a *Primary Table Book*, which appears to have been published separately at first. In 1856 the primary book is advertised as *Davies Primary Arithmetic and Table Book. Davies Intellectual Arithmetic* was first copyrighted in 1838 and recopyrighted in 1854, 1862, 1881. The practical arithmetic was first published in 1833 under the title *Common School Arithmetic*. In 1838 it was "enlarged and improved" and called *Arithmetic Designed for Academies and Schools*. In the preface of this edition Davies describes the book as an "elementary treatise." In 1848 this was revised and called *Davies' School Arithmetic*, and in 1855 another revision changed the title to *Davies' New School Arithmetic*. Later a "New Series of Arithmetics" was prepared, and the School Arithmetic became *Practical Arithmetic* and a new work, *Elements of Written Arithmetic*, was added to the series. The first edition of the *University Arithmetic* was published in 1846. It passed through many editions and was often revised. Greenwood says: "Whenever the discovery of new methods of presentation demanded a revision, the publishers and authors at once complied."

In 1912, the following arithmetics by Charles Davies were listed by the American Book Company: Primary Arithmetic, Practical Arithmetic, Elements of Written Arithmetic, and University Arithmetic.

Davies prepared "a full analysis of the science of mathematics," and explained "in connection the most improved methods of teaching." This was published in 1850 under the title, *The Logic and Utility of Mathematics*. This was based upon the system of mathematical instruction which had been "steadily pursued at the Military Academy (West Point) for over a quarter of a century." In describing this "system of mathematical instruction," Davies says:

> It is the essence of that system that a principle be taught before it is applied to practice; that general principles and general laws be taught, for their contemplation is far more improving to the mind than the examination of isolated propositions, and that when such principles and such laws are fully comprehended, their applications be then taught as consequences or practical results.
>
> This view of education led, at an early day, to the union of the French and English systems of mathematics. By this union the exact and beautiful methods of generalization, which distinguish the French school, were blended with the practical methods of the English system.[1]

And he sums it up by saying:

> And in that system (at the Military Academy) Mathematics is the basis; Science precedes Art; Theory goes before Practice; the general formula embraces all the particulars.

This system was the basis of Davies's arithmetics. In them, arithmetic is first a science.

In estimating the work of Charles Davies, Greenwood says:

> The influence of Dr. Davies's writings on subsequent authors in this country can hardly be overestimated. It may be very properly regarded as the beginning of a revolution in schoolbook making. Simplicity and extreme clearness became the leading ideas in the minds of authors, who studied how to be understood by children and young people.[2]

Joseph Ray (1807–1855) entered the Ohio Medical College in Cincinnati in 1828, graduated, and became a surgeon. In 1831 he became a teacher in Woodward College and professor of mathematics in 1834. This position he held until the institution was changed to Woodward High School in 1851, when he became president.

Ray's primary book was first published in 1834 with the title, *Ray's Tables and Rules in Arithmetic* and sold for 6 cents. In 1844 it was remodeled and became *Part First*, of Ray's Arithmetical Course. Since then it has been revised in 1853, 1857, 1877, 1903, and has appeared under several titles. In 1857, it was called *Primary Lessons*, in 1877, *Ray's New Primary Arithmetic*. The intellectual arithmetic was first published in 1834 under the title, *The Little Arithmetic; Elementary Lessons in Intellectual Arithmetic, on the Analytic and Inductive Method of Instruction*. In 1844, it was enlarged and called *Ray's Arithmetic, Part Second;* in 1857 it was

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[1] Preface.

[2] James M. Greenwood and Artemas Martin, "Notes on the History of American Textbooks on Arithmetic," Rept. U. S. Commis. of Ed., 1897–98, p. 839.

known as *Intellectual Arithmetic, by Induction and Analysis;* and in 1877 as *Ray's New Intellectual Arithmetic.* Under this last title it was copyrighted in 1905. *Ray's Eclectic Arithmetic on the Inductive and Analytic Methods of Instruction* was first published in 1837. In 1844 it was "carefully revised" and called *Ray's Arithmetic, Part Third,* and in 1857 it was again revised and called *Practical Arithmetic by Induction and Analysis.* In 1877 it became *Ray's New Practical Artithmetic.* In 1879, a two-book series was issued, *Ray's New Elementary Arithmetic* and *Ray's New Practical Arithmetic.* This series was revised in 1903 and the word "new" changed to "modern." *Ray's Higher Arithmetic* was published in 1856, the year after Dr. Ray's death. The text was completed and edited by Prof. Charles A. Mathews. It was revised and called *Ray's New Higher Arithmetic* in 1880.

Of all the texts of this period, the series by Joseph Ray has enjoyed the most extended and continued use. Ray's arithmetics became popular soon after their first publication in 1834, and it seems that their popularity increased rapidly for a number of years. Until within the last quarter of a century, no arithmetics were published which supplanted them except locally. Even now (1913), after more than a decade which has been characterized by texts of another type, they are still a widely used series of arithmetics. The average yearly sale for the last ten years has been approximately 250,000 copies.

J. M. Greenwood sums up his estimate of Joseph Ray and his work in these words:

> To many it has appeared strange why Ray's arithmetics have such a hold on the popular mind. The reason is, I think, obvious. Dr. Ray was, in a large sense, a self-made mathematician and a self-made teacher. He had learned well the lesson of self-help, and in the preparation of his books he always kept before himself all the difficulties he had experienced in mastering each topic. No one knew better just when and where and how to bear down on certain points. In an eminent degree he possessed that rare combination of assimilation and clear presentation. He knew how to make the subjects stick.[1]

Benjamin Greenleaf's (1786–1864) first book, the *National Arithmetic, Combining the Analytic and Synthetic Methods,* was published in 1835. Greenwood says: "It soon became a favorite treatise with teachers who preferred sound attainments in this science. The first edition was exhausted within a year." It was revised in 1836, 1847, and 1857, but the "general plan of the work was never changed." In 1836 it was announced as "Forming a complete mercantile arithmetic, designed for schools and academies." The edition of 1857 has the title, *The National Arithmetic, on the Inductive System, combining the Analytic and Synthetic Methods; Forming a Complete Course of*

[1] Op. cit., p. 840.

Higher Arithmetic. The *Common School Arithmetic or Introduction to the National Arithmetic*, was first published in 1842 and revised in 1848 and 1856. It is modeled closely after the *National Arithmetic*. *A Mental Arithmetic on the Inductive Plan* was first published in 1851 and revised in 1857 and 1863. The *Primary Arithmetic* was first published in 1857 and was revised in 1861.

In 1878 there were advertised Greenleaf's New Comprehensive Series, and Uniformity Arithmetical Series. The former consisting of the *New Primary Arithmetic*, *New Elementary Arithmetic*, and *New Practical Arithmetic*; the latter of *Greenleaf's New Primary Arithmetic*, *Greenleaf's New Intellectual Arithmetic*, *Greenleaf's Common School Arithmetic*, and *Greenleaf's National Arithmetic*.

About this time there appeared a series described as being "on the basis of the work of B. Greenleaf" which was copyrighted by H. B. Maglothlin, who assisted Mr. Greenleaf in revising the National Arithmetic in 1857. The texts in this series are: *First Lessons in Numbers, Oral and Written*, 1881; *Manual of Intellectual Arithmetic*, 1877; *Brief Course in Arithmetic*, 1881; and *New Practical Arithmetic*, 1876.

In 1912 Greenleaf's arithmetics were published by Benj. Sanborn Co., under the titles, *First Lessons*, *Complete Arithmetic*, and *Brief Course in Arithmetic*.

In the preface to the National Arithmetic, 1835, Greenleaf criticizes the system of arithmetic proposed by Colburn.

> An opinion has prevailed among some teachers that the pupil should have no rule to perform his questions by, but should form all his rules himself, by mere induction. This plan might do very well could it be carried into effect. But if the experience of the author has been of any service to him, one thing it has taught him is that, in a given time, a student will acquire more knowledge of arithmetic by having some plain rules given him, with examples, than he will without them, especially if he be required to give an analysis of a suitable number of questions under each rule.

He also finds it necessary to explain why he retains such topics as practice, progressions, position, permutations, etc., "which some arithmeticians of the present day, have laid aside as useless." His reason is:

> For though some of these rules are not of much practical utility, yet, as they are well adapted to improve the reasoning powers, they ought not, in the author's judgment, to be laid aside by any who wish to become thorough arithmeticians.

In a later edition he states that chief among the "many improvements on former editions" are "clearer definitions, more rigid analyses, and briefer and more accurate rules."

James Bates Thomson (1803–1883) wrote a series of arithmetics which were among the most popular during the middle of the century. The books of the series were: *Practical Arithmetic*, 1845; *Mental Arithmetic*, 1846; *Higher Arithmetic*, 1847; *Table Book*, 1848;

Rudiments of Arithmetic, 1852; and *Arithmetical Analysis*, 1854. In addition there was a *Commercial Arithmetic*, in 1884. The Practical Arithmetic is described by Greenwood as being "one of the best arithmetics ever offered to the public." All of the texts passed through many editions. It was stated in 1859 that 100,000 copies of Thomson's arithmetical works were circulated annually.[1] A new series in two books, *First Lessons in Arithmetic, Oral and Written Arithmetic*, and the *Complete Graded Arithmetic, Oral and Written*, was prepared by Mr. Thomson and published in 1882, just prior to his death.

John F. Stoddard (1825–1873) published his first arithmetic in 1849, and we are told "that up to 1860, 1,500,000 copies [of his arithmetics] had been issued, and the annual sales exceeded 200,000."[2]

The *Juvenile Mental Arithmetic*, 1849, was designed to be an introduction to the *American Intellectual Arithmetic* which was published the same year. The first was revised in 1857 and the latter in 1866. But these revisions were made "without any changes which might interfere with its use in the same classes with previous editions." There was another revision of the American Intellectual Arithmetic in 1889. The title was changed to Stoddard's *New Intellectual Arithmetic* and the book contains much new matter. Stoddard's *Practical Arithmetic*, 1852, was revised in 1865 and became the *New Practical Arithmetic*. Another revision was made in 1868. His arithmetical series also included *Pictorial Primary Arithmetic; Rudiments of Arithmetic*, 1862; *Complete Arithmetic; School Arithmetic*, 1869; *Ready Reckoner*, 1851; *Philosophical Arithmetic*, 1853.

Stoddard particularly emphasized mental or intellectual arithmetic. He says in the preface of the American Intellectual Arithmetic, dated 1860:

> That intellectual arithmetic, when properly taught, is better calculated than any other study to invigorate and develop the reasoning faculties of the mind, to produce accurate and close discrimination, and to enable the pupil to acquire a knowledge of the higher mathematics with greater ease, can scarcely admit of a doubt.

Greenwood says of the New Intellectual Arithmetic, "This book is one of the very best mental arithmetics published." The rule which the author says he has followed is, "Tell but one thing at a time and that in its proper place."

James Stewart Eaton (1816–1865).—The series of arithmetics by James Stewart Eaton consists of *High School Arithmetic*, 1857; *Primary Arithmetic*, 1860; *Common School Arithmetic*, 1863; *Intellectual Arithmetic*, 1864; *Elements of Arithmetic*, 1863. In 1879 Eaton's arithmetics were revised by W. F. Bradbury and published under the titles, *Bradbury's Eaton's Elementary Arithmetic*, in two

[1] Op. cit., p. 847.

[2] Op. cit., p. 852.

parts, and *Bradbury's Eaton's Practical Arithmetic*. The Common School Arithmetic was advertised as being in print in 1912. I have a copy of the High School Arithmetic, dated 1873 but copyrighted 1857, which has this complete title, "A Treatise on Arithmetic, Combining Analysis and Synthesis, Adapted to the Best Mode of Instruction in Common Schools and Academies." Eaton describes well this text when he says in the preface:

> It has been the guiding principle to be clear, brief, accurate, logical. Subjects are arranged, first, with reference to their dependence, and, secondly, with reference to their importance and simplicity—the less difficult and more practical first, and the more intricate and less important afterwards.

The Primary Arithmetic is handsomely illustrated. Greenwood says of the book:

> This arithmetic was published by Messrs. Brown and Taggard in 1860. * * * The object-lesson method in primary instruction is very prominent in this book. The engravings represent the objects with which the child is familiar. The aim of the author is to lead the young pupil to a knowledge of the rudiments of numbers by the use of sensible objects rather than by uninterrupted drill on the tables. The book is simple in language, progressive in arrangement, and well adapted to elementary instruction.[1]

The Common School Arithmetic follows the same general plan as the High School Arithmetic. It differs from it in being more practical and designed to meet the wants of the higher grades of common and grammar schools. In the Intellectual Arithmetic, Eaton attempts to build upon the Pestalozzian method. In the preface he says:

> The Pestalozzian or inductive method of teaching the science of numbers is now universally approved by intelligent teachers. The first attempt in this country to apply this method to mental arithmetic resulted in the publication of Colburn's First Lessons, a work whose success has not exceeded its merits. It was, however, a useful experiment rather than a perfect realization of the inductive system of instruction. That the subsequent books of the same class and purpose have failed to correct its defects, and thus meet the demand it created, is due evidently to their departure from the true theory as developed and exemplified by Pestalozzi. The author of this work has endeavored to improve upon all his predecessors by adhering more closely than even Colburn did to the original method of the great Swiss educator and by presenting at the same time, in a practical and attractive form, such improvements in the application of his principles as have stood the test of enlightened experience.
>
> In the spirit of the inductive method concrete numbers—numbers applied to physical objects—have been largely employed in treating of each topic, as the only fit preparation for the exercises upon abstract numbers, which are far more difficult for the youthful mind to grasp.

In this text the addition facts are in this form, "Two and one are how many?" and the general organization of the book is similar to that of Colburn's First Lessons.

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[1] Op. cit., p. 865.

Horatio N. Robinson (1806–1867) seems to have been a mathematical genius. At the age of 16 he made the astronomical calculations for an almanac. At the age of 19 he became professor of mathematics in the Naval Academy, which place he filled for 10 years. His first mathematical book was a *University Algebra* in 1847. Following this he wrote an *Elementary Algebra*, 1847; *Natural Philosophy*, 1848; *Elementary Astronomy*, 1849; *University Astronomy*, 1849; *Geometry containing Trigonometry and Conic Sections*, 1850; *Surveying and Navigation*, 1852; *Concise Mathematical Operations: A Practical Sequel to Mathematics and Astronomy*, 1854.

Robinson's series of arithmetics includes numerous titles. Greenwood gives the following list:

> Progressive Primary, 1858, Progressive Intellectual, 1858, Progressive Practical, 1858, Progressive Higher, 1860, Progressive Table Book, 1862, Rudiments of Written Arithmetic, 1858, First Lessons in Mental and Written Arithmetic, 1870, Junior Class Arithmetic, 1874, Elements of Arithmetic, Oral and Written, 1877; New Table Book, 1880; Robinson's Shorter Course, First Book, 1874, Beginner's Book, 1886; Shorter Complete Arithmetic, 1874; First Book (special edition), 1884.[1]

Although many of these texts passed through several editions and have been revised, only very few changes have been made from the original forms. For example, in a copy of the *Rudiments of Written Arithmetic* which bears the copyright dates of 1858, 1863, and 1877 it is stated that "there has been no change from previous editions." A similar statement is made in a copy of the *Progressive Practical* which bears the same copyright dates.

The First Lessons, Junior Class Arithmetic, Elements of Arithmetic, and New Table Book were special adaptations to meet local needs. A special edition of the First Lessons was issued for the St. Louis public schools in 1884. The Robinson series of arithmetics is at present published by the American Book Co. As advertised in 1912, the series consisted of 16 titles.

Daniel W. Fish assisted Mr. Robinson in the preparation of his texts, particularly in the revisions. In 1883 Fish published a two-book series, *Number One* and *Number Two*. It was stated that these belonged to the Robinson series, and they were of the same general type.

In speaking of the Progressive Intellectual Arithmetic, Robinson says:

> The arrangement and classification are more strictly systematic and in accordance with the natural order of mathematical science.
>
> One of the most important and, it is thought, one of the most original and useful features of this work is the full, concise, and uniform system of analysis it contains, the result of long experience in the schoolroom.

[1] Op. cit., p. 859.

In the preface to the Progressive Practical Arithmetic he states that his purpose has been—

> to present the subject of arithmetic to the pupil more as a science than an art; to teach him *methods of thought*, how to *reason*, rather than *what to do*; to give unity, system, and practical utility to the science and art of computation.

These statements are descriptive of Robinson's arithmetics.

Edward Brooks is another arithmetician whose original texts were adapted to conditions without very much change and appear in several forms. Originally the series consisted of *Normal Primary Arithmetic*, 1859; *Normal Mental Arithmetic*, 1858; *Normal Elementary Written Arithmetic; Normal Written Arithmetic*, 1863. This series was revised about 1875 and became *New Normal Primary Arithmetic, New Normal Elementary Arithmetic, New Normal Mental Arithmetic, New Normal Written Arithmetic*. In 1878 in the Normal Union Series oral arithmetic was united with written arithmetic. This appears to have been done to satisfy popular demand, and Mr. Brooks himself was undecided as to which was the better plan. After discussing the advantages and disadvantages of a union series, he says.

> What will be the final adjustment of this matter it is difficult to decide. The present tendency for combination is an example of history repeating itself. Soon after the method of arithmetical analysis, now taught in mental arithmetic, was presented, several authors published textbooks combining mental and written exercises, among whom may be mentioned Emerson and Roswell C. Smith. These books were very popular for a while, but the public taste changed, and the two subjects became separated, and mental arithmetic took its place alongside of written arithmetic, and has maintained it for many years. At present there is a demand for the combination of the two in one book. Whether this demand will be permanent or, like a new fashion, will change again in a few years, time alone can decide.[1]

Mr. Brooks published another series, *Normal Rudiments of Arithmetic*, and *Normal Standard Arithmetic*. These are advertised as "two entirely new books embodying Dr. Brooks's lifetime experiences in common-school work." Brooks's *Normal Higher Arithmetic*, 1877, was designed "to make the student a master of the theory of arithmetic." In addition Mr. Brooks wrote *Methods of Teaching Arithmetic* and *Philosophy of Arithmetic*. The former was printed with the *Key to Union Arithmetic*, and portions of it were reprinted in his texts. Brooks's arithmetics are still published (1912) by Christopher Sower Co.

Brooks's conception of arithmetic was essentially the same as that of the authors we have mentioned in the preceding pages. Arithmetic was primarily a science whose function was to discipline the pupil, and this was best accomplished by making the subject logical, concise, and scientific.

Other Important Arithmetics.[2]—The first four texts we shall mention were published in the decade immediately following 1821. Their

[1] Key to Union Arithmetic, p. 9.

[2] For a complete list of the texts of this period see J. M. Greenwood and Artemas Martin; "Notes on the History of American Textbooks on Arithmetic," Rep. U. S. Commis. of Educ., 1897-98 and 1898-99.

authors stated explicitly, either in the preface or the title, that the text was based upon Pestalozzian principles.

William B. Fowle, The Child's Arithmetic or the Elements of Calculation in the Spirit of Pestalozzi's Method, for the Use of Children between the ages of Three and Seven Years, 1826.

Roswell C. Smith, Practical and Mental Arithmetic on a New Plan, in which Mental Arithmetic is combined with the use of the slate; containing a complete system for all practical purposes; being in dollars and cents; 1827.

Martin Ruter, The Juvenile Arithmetic and Scholar's Guide; wherein Theory and Practice are combined and adapted to the Capacities of Young Beginners; containing a due proportion of examples in Federal Money; and the whole being illustrated by Numerous Questions similar to those of Pestalozzi, 1827.

James Ryan, The Pestalozzian System of Arithmetic, 1829.

Adams's New Arithmetic, by Daniel Adams, author of the *Scholar's Arithmetic*, was published in 1827. It is described on the title page as being a text "in which the principles of operating by numbers are analytically explained and synthetically applied; thus combining the advantages to be derived from both the inductive and synthetic modes of instructing."

The *New Federal Calculator*, by Thomas T. Smiley, 1828, is described as being "in appearance a twin to Daboll's Arithmetic." It passed through several editions and was still printed in 1899 by J. P. Lippincott & Co. The extended use of this text shows an element of conservatism.

Elementary Lessons in Intellectual Arithmetic, by James Robinson, 1830, was designed as an "introduction to Colburn's First Lessons and other arithmetics." The number facts are presented objectively.

Peter Parley's Method of Teaching Arithmetic, by S. G. Goodrich, 1833, is an interesting primary arithmetic.

George Perkins published *Higher Arithmetic*, 1841; *Primary Arithmetic*, 1850; and *Practical Arithmetic*, 1851. George P. Quackenbos builded upon the texts by Perkins in 1863 and following. Many of the problems in the Practical Arithmetic are made up of important statistics and valuable facts in history and philosophy.

A series of arithmetics was published by Horace Mann and Pliney E. Chase: *Elements of Arithmetic*, Part First and Part Second, 1850; and *Arithmetic Practically Applied*, 1850. In addition Mr. Chase published the following under his own name: *The Good Scholars Easy Lessons in Arithmetic*, 1845; the *Elements of Arithmetic*, 1844; and *Common School Arithmetic*, 1848. The last two of these are specified as being "on the plan of Pestalozzi," and the series is sometimes called a Pestalozzian series.

George A. Walton, assisted by Electa N. L. Walton, published a series of arithmetics in the sixties. Later in 1878 and 1884 George A. Walton with Edwin P. Seaver wrote the Franklin Arithmetics, which embody Pestalozzian ideas. A feature of both these series of arithmetics is the provision for drill. A separate book, *Arithmetical Problems*, 1872, by George A. Walton, contains over 12,000 problems for drill.

A series by S. A. Felter was first published in the years from 1862 to 1877. A prominent feature is the provision for drill.

The content and organization of the texts.—The series of arithmetics by Joseph Ray has probably been the most popular and the most extensively used texts of this period. The series is also the most representative of the content and organization of the texts of this period. In the following description we shall follow Ray's arithmetics, quoting from others only to emphasize a trait or to show the presence of a tendency which later modified the subject.

Primary arithmetic.—Warren Colburn intended his First Lessons to be a first text for a pupil, but it seems that, despite the very simple beginning, pupils found the book very difficult.[1]

The Child's Arithmetic, by W. B. Fowle, 1826, is a little volume of 104 pages. He states in the preface "that this manual is prepared in the spirit of Pestalozzi's method, and is intended as an introduction to the more advanced work of Colburn, which has wrought such a revolution in our own schools." The book is in three parts. The first has to do with numbers from 1 to 10, the second with numbers 10 to 20, and the third from 20 to 100. The first lessons contain explicit instructions for teaching children to count by using objects. They are taught to count out many of the number facts before they are given any practical examples. The practical examples are very similar to the simpler ones in Colburn's First Lessons. The book is a teacher's manual rather than a pupil's text. The plan is for the teacher to take the initiative; the pupil is to do what he is told to do. On the whole the book possesses no distinctive merit.

Emerson's North American Arithmetic, Part First, 1829, was a text in which "illustrations by the use of cuts" is made a very conspicuous feature in an attempt to exemplify the object teaching of Pestalozzi. Pictures of various objects are used—apples, cherries, trees, pears, hats, lamps, houses, horses, chairs, fishhooks, pins, etc. In many cases simply marks or stars are used. All concrete problems in the book, except miscellaneous problems, are graphically represented. Not all of the number facts are developed in this way, but such as are, always precede the formal statement and drill. This makes the form of the book inductive, although it is not noticeably so in spirit. The Hindu numerals are introduced in the very beginning and are used in stating the problems. The pages of the book are attractive in appearance and doubtless appealed to the child. On the whole it is a primary arithmetic of considerable merit.

[1] Clifton Johnson, Old Time Schools and School Books, p. 37.

The relation of this text to Pestalozzi is given by Emerson in the preface. He says:

The plan of the lessons accords with the method of instruction practiced in the school at Stans, by the celebrated Pestalozzi. The method of illustration, by the use of cuts, and the location of unit marks under the question, it is hoped, will be found to be an improvement.

The book was evidently designed to be used before a pupil commenced such a book as Colburn's First Lessons, since it was introduced immediately into the Boston schools, apparently without displacing the First Lessons which was then in use.

Peter Parley's Arithmetic is a quaint little volume. Its lessons are "About dogs," "About soldiers," "About money," "About a baker's shop," etc. Each lesson is headed by an appropriate picture. The following lesson, "About a cat and her kittens," is typical:

Here is a cat with four kittens. She has been out in the field where she has caught a bird; this she has brought home and given to the kittens. She has also caught a mouse, and one of the kittens is playing with it. Puss is a sly creature, and she kills a great many little birds and mice. Her foot is so soft that she can walk without noise, and her eye is so formed that she can see as well in the night as in the day. When all my little readers are asleep, she steals forth into the meadow or the wood, and woe to the mouse or bird that falls in her way.

1. If 1 cat kills 2 birds in a day, how many will 3 cats kill? 4? 5? 6?
2. If 5 kittens eat 2 mice in a day, how many will 10 kittens eat?
3. If a cat divides 4 birds between 2 kittens, how many will each kitten have?
4. If a cat kills 3 birds in a week, how many will she kill in 2 weeks? 3 weeks? 5 weeks? 5 weeks? &c.
5. If a cat kills 7 birds and mice in a week, how many will she kill in 14 days? 4 days? 4 days? &c.
6. If one cat kills 5 mice in a week, another 3, another 7, another 4, and another 2, how many do they all kill?
7. If 4 kittens have devoured 16 mice and 12 birds in a month, how many has each devoured?
8. If there are 21 mice in a house, and a cat kills 17 of them, how many are left?
9. If there are 18 mice in a barn, out of which a weasel kills 7 and a cat 11, how many are left?

In Elementary Lessons in Intellectual Arithmetic, by James Robinson, 1830, the illustrations consist of a figure 1 placed in small squares. These "are designed to be used as counters, in performing practical questions." Every fundamental number fact is illustrated in this manner.

A little later the primary texts came to conform to a rather fixed type which embodied many of Colburn's ideas. The problems were very simple and about things from the pupil's life. They were to be solved by means of objects and in the mind. There were no rules or definitions. Usually the texts were illustrated by means of cuts. Ray's Arithmetic, Part First, was advertised in 1843 as containing "very simple lessons for little learners, illustrated with amusing pictures, as cats, dogs, rabbits, boys, girls, etc." In another place reference is made to it being "illustrated with about 1,000 pleasing pictorial counters." The content of these primary texts sometimes included some of the more common and simple tables of denominate numbers. During the latter part of this period their content was increased, and they were made more formal. Illustration by cuts disappeared. In Ray's New Primary Arithmetic, 1877, there are only four pictures to illustrate the problems of the text, and all the tables of denominate numbers are given except those obsolete. Piece-meal treatment of the fundamental number facts was the characteristic feature of the general organization. For instance, in multiplication an entire "lesson" was devoted to the table of two's, another to the three's, etc.

Mental arithmetic.—Colburn's First Lessons was the pioneer in this field. The mental, or intellectual, arithmetics by other authors were patterned closely after this prototype. The oral arithmetic of Part Second of Emerson's North American Arithmetics is commensurate with the First Lessons and we find much similarity. The main differences are: Hindu numerals are used from the beginning, the traditional order of topics is followed, and some use is made of pictured objects for illustration, especially in the presentation of fractions. The book is inductive in form as well as in spirit, being very similar to Colburn's in this respect. As in the case of the First Lessons, a number relation is given first in a practical problem and is followed by the same combination in abstracted form. For example:

1. A lady divided 15 peaches among some little girls, giving 3 to each girl. How many girls were there?

Solution. As many times as 3 peaches are contained in 15 peaches, so many girls were there.

2. If you had 16 cents to lay out in pencils, and the price of the pencils was 4 cents apiece, how many could you buy for all the money?

3. How many times is 4 contained in 16?

4. If 4 horses are required to draw one wagon, how many wagons might be drawn by 20 horses?

5. How many times 4 in 20? How many are 5 times 4?

What became Ray's Arithmetic, Part Second, was first published in 1834 under the title, The Little Arithmetic; Elementary Lessons in Intellectual Arithmetic on the Analytic and Inductive Method of Instruction. In the preface, dated March 1, 1834, he acknowledges his indebtedness to Pestalozzian influences by saying:

> So far as the plan of the work is concerned, we make few pretensions to originality; we tread in the footsteps of Pestalozzi, and shall rejoice if this work should be the means of making more extensively known the principles of the analytic and inductive method of instruction.

The book begins with numeration, and the numbers 1 to 10 are represented pictorially by means of apples. The Hindu numerals are given, along with their names. The numbers up to 100 are given before addition, but it is suggested that the numbers from 51 to 100 "may be omitted until the pupil has made some progress in addition." A table of 100 stars arranged in the form of a square is used in teaching the pupils to count. They "may also be used as counters, though the fingers are generally to be preferred." Addition is begun with such examples as:

James had one apple and his brother gave him one more. How many had he? Then 1 and 1 are how many?

James had two apples and his brother gave him one more. How many had he? Then 2 and 1 are how many?

There are 25 more questions of this type, and the suggestion is made that the teacher make up many more. The addition tables are then given and are followed by 5 pages of abstract drill and 14 practical problems. The section is closed with a statement of the definition of addition in question and answer form.

Subtraction, multiplication, and division are presented in the same general way. Fractions are introduced with the suggestion that "for illustration, the teacher should be provided with a number of apples." Halves and fourths are pictured as parts of apples, and the first problems are concerning apples. In the following lessons the teacher is advised to use other illustrative materials. The first lesson on fractions is made up of questions such as, "If you divide an apple into four equal parts, what is one part called? What are two parts called? How many fourths in one apple? In two apples? In three apples?" "How many fourths in one apple and one-fourth of an apple?" The next lesson takes up in order the fractions from halves to tenths in this manner.

If an apple be worth 3 cents, what is one-third of it worth? What is 2-thirds of it worth?

What is one-third of 3? What is 2-thirds of 3?

If an orange is worth 3 cents, what part of the orange will 1 cent buy? What part will 2 cents buy?

1 is what part of 3? *Ans.* 1 is the 1-third of 3.

2 is what part of 3? *Ans.* 2 is the 2-thirds of 3; that is, 2 is 2 times the one-third of 3.

If a yard of cloth cost 3 dollars, how much can you buy for 4 dollars? How much for 5 dollars?

4 are how many times 3? *Ans.* Once 3 and the third of 3.
5 are how many times 3? *Ans.* Once 3 and 2-thirds of 3.
6 are how many times 3? 7 are how many times 3?
8 are how many times 3? 9 are how many times 3?
10 are how many times 3? 11 are how many times 3?

Following this there are problems of division like "57 are how many times 5? 6? 7? 8? 9? 10?" and problems of multiplication such as "8 times 6 and 2-sixths of 6 are how many?" These two questions are then combined in one exercise and in later lessons there are examples which call for operations of the following types: "7 is one-fourth of what number?" "What is 2-thirds of 12?" "4-fifths of 25 are how many times 6?"[1] These operations are, in general, introduced by practical examples. Abstract examples are then given for drill, and more practical examples for application. At the end 59 miscellaneous problems are given.

The last section of the text is devoted to the tables of Federal money, dry measure, wine measure, Troy weight, apothecaries' weight, avoirdupois weight, long measure, cloth measure, square measure, measure of time, and sterling money. The plan of treatment is first the table and then questions for drill. Most of these are in the form, "How many quarts in 1 peck? 2? 3? 4?" Only a few of the questions approach practical problems.

In 1843 the Little Arithmetic was revised and enlarged and was published under the title "Ray's Arithmetic, Part Second." The first 54 pages are identical with the first 57 pages[2] of the Little Arithmetic except that the first problems in addition, subtraction, multiplication, and division are illustrated by small circles. Beginning on page 55, fractions are presented again more formally, but the author still retains much of the form and spirit of the earlier pages. The fractions are represented by dividing a "yard of tape." Pages 97 to 144 are given to "Practical Arithmetic." This includes notation and numeration up to nine places, the four operations for integers, reduction, and the four operations for denominate numbers, simple proportion, or the rule of three, and simple interest. In general, the presentation is formal, a single practical problem followed by the definition, explanation of the solution, statement of the rule, and abstract exercises for drill. Practical problems as applications are placed last.

In 1857 Ray's Arithmetic, Part Second, had the title, "Intellectual Arithmetic by Induction and Analysis." The important changes are "appropriate models of analysis and frequent reviews," the introduction of percentage, gain and loss, interest, and their applications, and the addition of a number of difficult problems. The "appropriate models of analysis" are given following the first problem of a lesson and again when a new type of problem is encountered. There is a tendency to place the abstract work before the practical problems. This is particularly true in the topics which have been added.

The edition of 1877 contains few significant changes. Objective illustrations are omitted. The presentation of fractions is more formal, the definition being given first and the logical order is approached. The space given to percentage and its applications is increased.

[1] Compare these exercises with the ones used by Pestalozzi. See p. 59. Ray followed Pestalozzi even more closely than did Colburn.

[2] These pages include all of the text except the section devoted to denominate numbers.

The content of the mental arithmetics followed very closely that of Colburn's First Lessons. The four operations for integers and for vulgar fractions, a few of the most important tables of denominate numbers, percentage, and interest would serve well as a table of contents for any mental arithmetic. The only change necessary would be in the order and emphasis.

The topics, as in the case of Ray's text, are presented in very much the same fashion as in the First Lessons. Each is introduced by practical problems, which are followed by abstract ones for drill. This order is almost invariably retained, even in revised editions and texts published well toward the close of this period. The plan of following a practical problem by the same combination with abstract numbers was not followed except during the active period, and the number of abstract drill exercises were relatively less during the static period. Much space was given to review questions, miscellaneous problems, and promiscuous examples. This was an evident attempt to secure thoroughness.

The practical problems are essentially of the same quality as those in Colburn's text. In fact, those of some of the texts bear a very close resemblance to those of the First Lessons. Toward the close of the period there is a noticeable increase in the difficulty of the problems. They were made more difficult in two ways: First, the magnitude of the quantities was made greater; second, the problems themselves were made intricate. The extent to which this was carried is shown in the following problems selected from Ray's Intellectual Arithmetic, one thousandth edition, 1860:

A hare is 100 leaps before a hound and takes 5 leaps while the hound takes 3, but 3 leaps of the hound equal 10 of the hare: how many leaps must the hound take to catch the hare?

A trout's head is 4 in. long, its tail is as long as its head and ½ of its body, the body is as long as its head and tail; what is its length?

If 10 gal. of water per hr. run into a vessel containing 15 gal. and 17 gal. run out in 2 hr., how long will the vessel be in filling?

A, B, and C rent a pasture for $92. A puts in 4 horses for 2 mon., B 9 cows for 3 mon., and C 20 sheep for 5 mon. What should each pay, if 2 horses eat as much as 3 cows and 3 cows eat as much as 10 sheep?

If the interest for 1 year 4 mon. is 3/25 of the principal, what is the interest of $100 for 1 yr., 8 mon., 18 da.?

The number and difficulty of such problems varied with the author. Ray probably represents an average, certainly not less than an average. Texts containing difficult problems seem to have been demanded, particularly in the latter part of this period. In his New Mental Arithmetic,' 1873, Brooks gives a large number of problems which are nothing more than intricate puzzles. He classifies them under such heads as pasture problems, beggar and equal number problems, animal problems, working problems, labor and fish problems, age and step problems, etc.

Practical arithmetic.—The texts which are grouped under this head included all the topics of arithmetic which were studied in the elementary school. They began with numeration and notation, and addition, subtraction, etc., came in turn. In scope they were the descendants of such texts as those of Dilworth, Daboll, and Adams. The study of "practical arithmetic" paralleled that of "mental arithmetic."

Colburn applied his ideas of arithmetic, particularly the inductive method, to this field and, as we have shown, produced a text of high merit. But the Sequel was not well received, and after a few years dropped out of notice. The book embodied some features, such as the entire departure from the traditional division of subject matter and the order of topics, which were too progressive for the times. Furthermore, the book had to compete with contemporary texts and texts which were already in use. This was not true of the First Lessons, for it was a pioneer in a new field. Thus Colburn probably influenced only very slightly this field of arithmetic directly through the Sequel. However, the work of Colburn did change the texts in practical arithmetic. The source of this influence was primarily the First Lessons. The principles underlying this text were accepted, and other writers, like Colburn, attempted to apply them in part to the more advanced texts.

We have described the Scholar's Arithmetic by Daniel Adams. In 1827, he says in the preface of Adams's New Arithmetic:

The Scholar's Arithmetic, published in 1801, is synthetic. If that is a fault of the work, it is a fault of the times in which it appeared. The analytic or inductive method of teaching, as now applied to elementary instruction, is among the improvements of later years. Its introduction is ascribed to Pestalozzi, a distinguished teacher in Switzerland. It has been applied to arithmetic, with great ingenuity, by Mr. Colburn, in our country.

The analytic is unquestionably the best method of acquiring knowledge; the synthetic is the best method of recapitulating, or reviewing it. In a treatise designed for school education both methods are useful. Such is the plan of the present undertaking, which the author, occupied as he is with other objects and pursuits, would willingly have forborne, but that, the demand for the Scholar's Arithmetic still continuing, an obligation, incurred by long-continued and extended patronage, did not allow him to decline the labor of the revisal, which should adapt it to the present more enlightened views of teaching this science in our schools. In doing this, however, it has been necessary to make a new work.

Division is introduced with the problem, "James divided 12 apples among 4 boys; how many did he give each boy?" After 20 problems of this sort, the problem, "How many oranges, at 3 cents each, may be bought for 12 cents?" is solved by successive subtractions. The pupil is then told, "We may come to the same result by a process, in most cases much shorter, called *Division*." The process of division is explained by solving this problem, and the new words are defined. The division table is given, and after the problem, "How many yards of cloth, at 4 dollars a yard, can be bought for 856 dollars?" is solved and explained, the rule is stated for the case when the divisor does not exceed 12. The rule for the case when the divisor exceeds 12 is derived in the same manner.

A comparison of this presentation of division with the way in which such topics were presented in the texts of the preceding period indicates the extent of Colburn's influence upon the "practical arithmetics." The inductive method was accepted with only slight reservation, and Adams seems to have caught something of the spirit of it, as well as the form.

Roswell C. Smith, whose arithmetic was first published in 1827, says in a rather bombastic preface in the third edition, 1834:

> Another inquiry may still be made: Is this edition different from the preceding? The answer is, Yes, in many respects. The present edition professes to be strictly on the Pestalozzian, or inductive, plan of teaching. This, however, is not claimed as a novelty. In this respect, it resembles many other systems. The novelty of this work will be found to consist in adhering more closely to the true spirit of the Pestalozzian plan; consequently, in differing from other systems, it differs less from Pestalozzian. This similarity will now be shown.

The author attempts to combine oral and written arithmetic. Certain features of the oral part of the text almost duplicate Colburn's First Lessons both in actual content and spirit. For example, Smith begins his text with:

> 1. How many little fingers have you on your right hand? How many on your left? How many on both?
> 2. How many eyes have you?
> 3. If you have two apples in one hand, and one in the other, how many have you in both? How many are two and one, then, put together?
> 4. How many do your ears and eyes make, counted together?
> 5. If you have two nuts in one hand, and two in the other, how many have you in both? How many do two and two make, put together?

The Hindu numerals are introduced on page 2 and the addition tables are given on pages 3 and 4. Aside from a list of 24 problems, there is no work preceding the tables, and these problems do not constitute a development of the addition facts. Following the table, there is only a page of drill. The remaining three operations are disposed of in the same manner. This completes the "mental exercises." Beginning on page 17 we find the traditional order of topics, numeration and notation, addition, etc. The fundamental operations are introduced by a list of practical problems to be solved mentally. The "interrogative system"[1] is used throughout the work in presenting rules and explanation. The author did this under the impression that it was the mark of inductive presentation, but nevertheless the subject is presented quite dogmatically. For the most part the spirit of the book is deductive rather than inductive.

[1] This same interrogative, or question and answer, system was used in Dilworth's Schoolmaster's Assistant. Therefore this feature is not necessarily due to Pestalozzi.

In presenting a new process, e. g., long division, one problem is solved and explained, after which the rule is stated.

The text, while it possesses some merit and must be considered one of the progressive texts of its time, does not reflect much of Pestalozzian principles and does not equal Colburn's texts in this respect.

The plan of the written arithmetic of Emerson's North American Arithmetic, Part Second, is much like the oral part. But the structure of the book is more formal, objective illustration is lessened, and the inductive method, although retained, has lost much of its spirit. The point of departure in taking up a new process is not always a concrete problem, and the development is forced. For example, in division:

> How many yards of cloth, at 3 dollars a yard, can be bought for 396 dollars?
>
> Here we must find how many time 3 dollars there are in 396 dollars: that is, we must divide 396 by 3. $\dfrac{3)396}{132}$ We first divide the 3 hundreds, then the 9 tens, and then the 6 units; thus, 3 in 3, once; 3 in 9, 3 times; 3 in 6, 2 times.
>
> Observe, in the above example, that the 3 which we first divide means 3 hundred; and the 1 which we place under it means 1 hundred, showing that 3 is contained in 300, 100 times. The 9 means 9 tens, and the 3 which we place under it means 3 tens, showing that 3 is contained in 90, 30 times.
>
> A *dividend* is a number which is to be divided, such as the number 396 in the above example. A *divisor* is a number by which we divide; such as the number 3 in the above example. The *quotient* is the number of times which the divisor is contained in the dividend; such as the number 132 in the above example.

Long division comes four pages later. The topic is introduced as follows:

> The method of dividing taught in the two preceding sections is called *short* division, the method taught in this section is called *long* division. In long division we place the quotient on the right hand of the dividend, and perform the same operations under the dividend, heretofore performed in the mind.

```
4)95307(23826   How many times is 4 contained in 95307?
  8
  --
  15
  12
  --
   33
   32
   --
    10
     8
    --
     27
     24
     --
      3
```

> Perceiving that 4 is contained in 9 twice, we place 2 in the quotient, multiply the divisor by 2, and subtract the product (8) from 9. This is the same as saying in short division, "4 in 9, 2 times and 1 over." Now, since the 1 over must be joined with the 5, we bring down the 5 to the right of the 1; and then, perceiving that 4 is contained in 15, 3 times, we place 3 in the quotient, multiply the divisor by 3, and subtract the product as before. Thus we proceed to bring down every figure of the dividend and unite it with the previous remainder.

Additional difficulties encountered in division are explained in the same manner, which is essentially only a detailed rule stated for a particular example. After 24 examples, all abstract, the general rule is stated. (Compare this with Colburn's presentation of division, page 74.)

Although Emerson presents percentage and interest as distinct topics, his treatment resembles that of Colburn in the Sequel. In the case of interest, the rule is stated for the simple case preceding the first problems and the additional rules are given as needed, sometimes even being given after the first problem which demands them. On the whole, Part Second is a text of considerable merit, although in inductive treatment, concreteness, and motive it is inferior to Colburn's text.

The text which came to be known as Ray's Practical Arithmetic begins with an introduction of 19 pages devoted to definitions, numeration, and notation. It is suggested that "pupils in general need not be required to study the introduction until going through the book the second time." Addition begins with:

If you find 2 cents at one time and 3 at another, how many will you have?

If you give 12 cents for a slate and 5 cents for a copy book, how much will they both cost?

John gave 6 cents for an orange, 7 cents for pencils, and 9 cents for a ball; what did they all cost?

After several problems of this sort the pupil is told that—

the putting together of two or more numbers of the same name or denomination, so as to make one number (as in the preceding examples), is called addition. The number formed by adding together two or more numbers is called the sum or amount.

The tables are then given, and following is this development of the rule:

James had 63 cents and his father gave him 35 more; how many cents had he then?

These numbers being too large to be added conveniently in the mind, it becomes necessary to write them down, and in doing this it is necessary to put the units of one number under the units of the other, and the tens of one number under the tens of the other, to enable us more easily to add together figures of the same local value.

	63 cents
	35 cents
Ans.	98 cents

Having written the numbers in this manner and drawn a line underneath, we begin at the right hand and add the 5 units of the lower number to the 3 units of the upper number, which makes 8 units; we write this in units' place, and then add the 3 tens of the lower number to the 6 tens of the upper number, which makes 9 tens to be set down in tens, place and the work is completed, and the sum of 63 cents and 35 cents is 98 cents.

James bought an eclectic reader for 72 cents, an arithmetic for 37 cents, and a slate for 9 cents; how much did they all come to?

E. reader,	72 cents
Arithmetic,	37 cents
Slate,	9 cents
Ans.	118 cents

In writing this example we put the 9 under the 7 units' in place, for if put under the 3 in tens' place it would count 9 tens, or 90 cents.

We begin at the right-hand column and say 9 (units) and 7 (units) are 16 (units), and 2 (units) are 18 (units), which is 1 ten and 8 units; we set down the 8 (units) in the units' place, and carrying the 1 (ten) to tens' place, we say 1 (ten) and 3 (tens) are 4 (tens) and 7 (tens) are 11 tens; that is, 1 hundred and 1 ten, which we put in their proper places.

The rule is followed by abstract drill exercises and then practical problems. Subtraction, multiplication, and division are similarly presented. Fractions are presented somewhat more formally, but the inductive form and much of its spirit is exhibited in the remainder of the book. The table of contents includes the four operations for simple numbers, Federal money, compound numbers, fractions, percentage and its applications, ratio and proportion (simple and compound), fellowship, alligation, equation of payments, practice, and analysis. In an appendix the following topics are presented briefly: Involution, evolution, progressions, position, permutation, exchange, duodecimals, mensuration, and bookkeeping.

Essentially the same introductory problems are found in the edition of 1857. Following these, the definitions are stated thus:

The process of uniting two or more numbers into one number is termed Addition.
The number obtained by addition is the Sum or Amount.
Remark.—When the numbers to be added are of the same denomination—that is, all cents, or all yards, etc.—the operation is called Simple Addition.

The development of the rule is as follows:

1. James had 63 cents and his father gave him 35 cents; how many cents had he then?

Place the units and the tens of one number under the units and the tens of the other, that figures of the same unit value may be more easily added.

Solution.—Write the numbers as in the margin; then say 5 units and 3 units are 8 units, which write in units' place; 3 tens and 6 tens are 9 tens, which write in tens' place. The sum is 9 tens and 8 units, or 98 cents.

63 cents
35 cents
Ans. 98 cents

In this example, units are added to units, and tens to tens, since only numbers of the same kind—that is, having the same unit value—can be added. Thus, 3 units and 2 tens make neither 5 units nor 5 tens; as, 3 apples and 2 plums are neither 5 apples nor 5 plums.

This is followed by a second example and its solution, and then a few "questions to be solved as above." Carrying is avoided in these examples and is taken up on the next page, after which the rule is stated.

A comparison of this presentation of addition with that in the first edition (1837) shows the extent of the formalization of the inductive method which was attained in Ray's Practical Arithmetic by 1857. The form is retained in part, but the spirit of it is almost entirely lost. In the derivation of the rule the pupil is simply told what to do. It differs only in form from the method of presentation found in Adams' Scholar's Arithmetic (1801).

In the next revised edition, 1877, there is one additional introductory problem, and the definitions are stated more formally. Only two problems are solved preceding the statement of the rule, as against four in the edition of 1857. And one of these is purely abstract. The explanations of the solutions are more abbreviated and more dogmatic.

In the first edition the topics which we now include under the head of percentage and its application are given in the order, simple interest (including partial payments), banking, discount, percentage (including profit and loss), commission, insurance, buying and selling stocks, exchange, duties, and taxes. Under interest, rate is introduced by the statement: "A lent B $200 for one year; at the end of the year B paid A the $200 which he had borrowed and also $12 in addition for the use of the money; that is, he paid at the rate of $6 for the use of $100 for one year." From this, principal, interest, rate per cent, and amount are defined. "Rate per cent means rate per hundred; it is the sum paid for the use of one hundred dollars for one year." Later, per centage (written as two words) is defined as the group of "those calculations in which reference is made to a hundred." Per cent is defined as a "contraction of per centum, which signifies by the hundred; thus when we say 5 per cent, we mean 5 dollars on 100 dollars, or 5 cents on 100 cents."

Decimal fractions are not employed in these problems. The rule "to find the per centage" is: "Multiply by the rate per cent and divide by 100; the quotient will be the per centage." After the pupil has been drilled upon finding the "per centage," the application is made to profit and loss. This plan is followed for each of the three cases of percentage.

The presentation of this material is practically unchanged in the edition of 1844, but in 1857 percentage (written as one word) was first presented abstractly (Cases I and II). This is followed in order by commission, insurance, stocks, brokerage, interest, partial payments, compound interest, discount (true), bank discount, profit and loss, taxes, and duties. In presenting percentage, the equivalent of common fractions in terms of per cents is given first. In solving the problems the rate per cent is to be expressed decimally. The symbol "%" is used almost exclusively instead of the words "per cent." The idea of "per cent" meaning at the rate of so much on the hundred is not suggested.

In 1857, common fractions are preceded by sections devoted to factoring, greatest common divisor, and least common multiple. Longitude and time is made the title of a section. Aliquots, or practice, is reduced to a scant three pages, and cancellation is introduced, but Ray insists that "it is not made a *hobby*, or an arithmetical machine, by which results can be obtained merely in a mechanical manner."

A comparison of the editions of 1837 and 1877 reveals the following changes of content. The Roman notation, cancellation, and the metric system have been added. Factoring (including least common multiple and greatest common divisor), complex fractions, percentage, exchange, insurance, and taxes have been enlarged. The following topics have been omitted: Alligation medial, single position, double position, permutation, duodecimals, bookkeeping, and obsolete tables of weights and measures. Some of the modifications are very noticeable. For instance, in 1837 exchange was given a scant half page in the appendix. In 1877 it was made a chapter of five pages. In 1837 percentage was treated briefly, following interest. In 1877 a chapter of 22 pages is given to percentage, and it precedes interest.

In the National Arithmetic by Benjamin Greenleaf, 1835, mental arithmetic is mixed with the written by giving a list of "mental operations," but young pupils are advised to study Colburn's First Lessons. The mental problems precede the written, and usually the first of the mental are abstract. "Mental operations in fractions" reminds one of Colburn's First Lessons. In this list the practical examples precede the abstract. Interest is the first topic of percentage. The rule is, "multiply by the rate per cent and divide by 100." A second rule is to express the per cent decimally. Percentage as a topic does not appear. Practice, proportion, and fellowship follow interest, commission and brokerage, stocks, and insurance. Later, taxes, banking, and loss and gain are given. The problems under loss and gain are to be solved by proportion, the rule of three.

In the edition of 1857 all the mental problems are omitted except those preceding fractions which are given under the head, "Examples to be performed by analysis." The text begins with a list of definitions and axioms. Cancellation is made a topic and is much used. Longitude and time is introduced. Fractions are introduced by a section called, "Properties of numbers." This contains factoring, greatest common divisor, and least common multiple. Ratio and proportion precedes percentage and its applications, although it does not appear that the pupil is expected to make use of proportion in solving the problems of percentage. The treatment of percentage and its application is very similar to that in the 1857 edition of Ray's Practical Arithmetic. It is abstract and formal. The per cent sign (%) is not used.

The problems in practical arithmetic.—In the texts of the ciphering-book period the major portion of the space was given to the rule of three, exchange, practice, etc. With the spread of the use of Federal money, practice and exchange were practically eliminated. The rule of three was minimized. On the other hand, "practical" problems were given as applications for the fundamental operations. Ray gives 24 abstract and 12 "practical" problems in addition, and approximately this ratio is maintained.

The word "practical" is used advisedly. Among those we included under the head "practical" are some of the following:

"How many lessons of 95 lines each are contained in Virgil's Aeneid, the number of lines in the poem being 9,892?

"Rees' Cyclopaedia consists of 39 volumes, each containing, at an average, 774 pages, of two columns. In each column there are 67 lines, each containing, at an average, 10 words, and in these 10 words, there are, at an average, 47 letters. Required the number of pages, lines, words, and letters, contained in the entire work.

The entire quantity of tea sold by the East India Company in 1799 was 24,853,503 pounds; how many chests, each containing 87 pounds, would this quantity fill?

This type of problems was not original with Ray, as such problems are found in much earlier texts. However, during this period they became more prevalent. Occasionally an author exhibited extreme tendencies. For instance in the Franklin Arithmetic, 1832, such problems as the following are found:

How many letters in the word Smith?

In eighteen hundred and thirty-one, 119 persons died of drunkenness in New York, and 137 in Philadelphia; how many in both?

Take E from the word HOPE, and how many letters would be left? and what would it be then?

Four rivers ran through the garden of Eden, and one through Babylon, how many more ran through Eden than Babylon?

The Baltimore railroad cars run 12 miles an hour; what is four sevenths of it?

A human body, if baked until all moisture is evaporated, is reduced in weight as 1 to 10; a body that weighs 100 pounds living will weigh how much when dry?

In an arithmetic by Horace Mann and Pliney E. Chase, published in 1850, this type of problem was made a feature. The idea was conceived by Mr. Mann, although to Mr. Chase is due most of the credit for its execution. In the preface of Arithmetic Practically Applied, Mann sets forth his ideas as follows:

Believing the idea of the work to be original, I will attempt its elucidation. In seeking for the elements or materials of its questions, it proposes to take a survey of all the truths of science, and to make a selection from each department of whatever may be most interesting and valuable. It does not confine itself to the playthings of the nursery, or to the commodities of the market place, and to the money they will cost, or make, or lose. On the contrary, the present work proposes to carry the student over the wide expanse of domestic and social employments; to introduce him to the various departments of human knowledge so far as that knowledge has been condensed into tables, or exhibited in arithmetical summaries, and to make him acquainted with many of the most wonderful results which mathematical science has revealed. Instead of groping along the mole-path of an irksome routine, with little other change than from dollars and cents to pounds and pence, or some other familiar currency, and with little other variety than from cloth to corn, or some other commonplace commodity, it derives its examples from biography, geography, chronology, and history; from educational, financial, commercial, and civil statistics; from the laws of light and electricity, of sound and motion, of chemistry and astronomy, and others of the exact sciences. Trades, handicrafts, and whatever pertains to the useful arts, so far as they are the subject of numerical statement, and their facts possess arithmetical relations, together with all the ascertained and determinate results of economical or political knowledge, and of scientific discoveries, are laid under contribution, and are made to supply appropriate elements for the questions on which the youthful learner may exercise his arithmetical faculties.

Two advantages are mentioned which Mann says "seem to me unquestionable:"

1. The pupil, while studying arithmetic for its own sake, will acquire some knowledge of many other things.

2. The work will appeal to more faculties of the mind, and hence afford opportunity for alternate use.

Greenwood says of the book:

The information composing the problems is drawn from at least a hundred sources. It is highly instructive as well as eminently practical. A good title to the book would be "Useful and Scientific Information Treated Arithmetically." A revised edition ought to be in the hands of every teacher. It has never been properly appreciated, and few copies are in existence; even the publishers do not have a copy.[1]

Organization.—In the internal arrangement many of the texts in the active period explicitly professed to be upon the inductive plan, and we have shown that the authors did build upon this plan with some degree of understanding of the mental process involved in induction. But it appears that about the middle of the century this understanding of the inductive plan faded or became overshadowed by philosophical considerations.

In the Practical Arithmetic of 1857 Ray says that the "inductive and analytic methods" are adopted. Two paragraphs later he states that "the arrangement is strictly philosophical; no principle is anticipated; the pupil is never required to perform any operation until the principle on which it is founded has first been explained." This is a contradiction, because induction and what he defines as the "philosophical arrangement" are fundamentally opposed. The text itself exhibits both plans of organization, but the spirit of the text is more in accord with the latter. In 1877 there is no reference to the "inductive and analytic methods." Ray's text does not represent an extreme in respect to this trait. Davies, Greenleaf, Brooks, Robinson, and others are more pronouncedly "philosophical."

Higher arithmetic.—The higher arithmetics were simply an advanced treatise on the general plan of the practical arithmetics. All topics were included from notation and numeration and the fundamental operations for integers to involution and evolution and series. The topics were not introduced by a few questions to be answered orally. The organization was strictly logical, first such definitions as were necessary, then the rule followed by abstract exercises, and finally practical problems. The subject was looked upon throughout the text primarily as a science. Emphasis was placed upon "clearer definitions, more rigid analyses, and briefer and more accurate rules." These features represent the prime merit of not only the higher arithmetics, but of the practical arithmetics as well. This is undoubtedly one of the main reasons for the long and extensive use of such texts in our schools.

Colburn's influence upon the textbooks of this period.—Primary texts, "mental arithmetic," the use of objective materials, and the inductive method were the most significant features of the arithmetics of

[1] Op. cit., p. 854.

the period. The authors of some of the texts were acquainted with Pestalozzi's system of arithmetic and his educational principles, but it is probable that all were acquainted with Colburn's texts, particularly the First Lessons, and the interest in Pestalozzi's system of arithmetic was due in a large measure to the popularity of this text. The primary texts were patterned after the First Lessons; the "mental arithmetics" followed it very closely; and the inductive method, before it was formalized, was very similar to that in Colburn's texts. The objective materials were changed only by the omission of the Pestalozzian tables and by adding pictures. Thus, much is due to Warren Colburn for stimulating and directing the development of American textbooks on arithmetic during this period.

Chapter IX.

TEACHING ARITHMETIC BY DEVICES AND DRILL.

The introduction of mental arithmetic, the concept of mental discipline as the function of arithmetic, the teaching of arithmetic to young children, the ideal of skill and thoroughness, the very great increase in the number of pupils studying arithmetic, and other changes necessitated modifications in the methods of teaching arithmetic. These modifications are described under the following heads.

Class instruction.—In view of the fact that in the ciphering book method, the individual contact between the teacher and pupil was for examining the pupil's work and telling him whether it was right or wrong, and not for instructing the pupil, much time was wasted. Before 1821, the monitorial system of instruction had been applied to arithmetic, and after this date class instruction in arithmetic was the rule. This was probably because it was more economical, but some teachers believed that in a group superior instruction was possible. For example, Ray says: "Pupils study best in classes; it is almost as easy for a teacher to instruct 15 pupils in a class, as 1 alone."[1]

For a teacher to handle a group of pupils successfully, some technique was necessary, and much attention was given to this phase of teaching during this period. What was written on the teaching of arithmetic was confined almost wholly to the elaboration of the technique of class instruction. The following report of the Boston Monitorial School gives a good description of one type of class instruction.

The next exercise is arithmetic. I have already said that even the youngest is taught to count and perform simple operations with beans, her fingers, and such aids. Soon a little mental arithmetic is introduced; but, as the excellent little work of Colburn is too difficult for such small children, manuscript questions prepared by the instructor are used. Next, Colburn's First Lessons are studied; and about the same time, written arithmetic is gradually introduced. This, however, is for the present completely subordinate to the intellectual. The monitors of arithmetic recite to the master, and then disperse to their stations to act as monitors. Their classes form around them; and the lesson which has been previously set is recited. If any explanations are necessary, the monitor, who has gone over the ground before, explains; but if she is at a loss, she applies directly to the master. In this way, the little classes get a great deal of practice, and the monitor reviews her studies. For the sake of variety, they then take slates and cipher. The monitor dictates sums verbally, and

[1] Key to Ray's Practical Arithmetic, p. 177.

the children are taught to write amounts from dictation. They are never allowed to copy sums, and consequently must acquire a knowledge of *numeration*, as useful as it is uncommon. In addition the highest adds the first column aloud and tells the next what to set down and what to carry; the next takes the second column, and does the same. Anyone who corrects another goes above her, as in spelling or reading; and, as all must aid in doing the sum, the attention of all is secured. It is so with subtraction, and all the other rules. The highest scholars cipher in Colburn's Sequel, and record their operations in a manuscript.[1]

The monitorial plan of group instruction was not generally adopted. Ray describes the practice of about 1840 as follows:

When practicable, the pupils should be arranged in classes, due regard being had to their ages, acquirements, etc. After this, the proceeding in the best schools, is somewhat as follows:

A certain number of examples is arranged as a lesson; it will, also, frequently be necessary that a part, or even the whole, of the lesson shall consist of the illustration of principles, or the memorizing of definitions or rules. When the class meets for recitation, each pupil passes his slate into the hands of the pupil next above him, except the pupil at the head, who passes his to the foot scholar. The teacher then reads the answer to the first question, while each pupil examines the slate he holds, to see if the answer is correct and properly obtained.

In addition to reading the answer, the teacher, in many cases, such, for example, as proportion, should state the general method of working the question. The pupils mark the answers that are wrong, or obtained improperly. In the same manner, each question is examined and marked. Instead of the teacher reading the answers, the pupils in succession may read them.

When there is a blackboard (and there should be one in every schoolroom, 4 or 5 feet wide, and as long as the room will permit), each pupil should be required to work out one or more of the examples, and give the reasons for performing the operation. The time required to examine the questions is generally short, while the habit of closely scrutinizing each other's work, improves the perceptive faculties of the pupils.[2]

William B. Fowle was the author of an arithmetic and for a number of years was an instructor in teacher's institutes in Massachusetts and New York. In the *Teacher's Institute* he discusses the teaching of the common branches. The following "methods" of teaching addition are interesting as well as typical:

When the children are ciphering on the blackboard, there are various ways of keeping them at work. I will try to describe a few of them. Suppose the class consists of six, and the exercise to be in addition. I first dictate one line of a sum to each pupil, as follows:

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3, 746, 389, 467
7, 680, 895, 089
8, 070, 688, 496
9, 009, 900, 090
7, 508, 785, 687
8, 687, 768, 686
              515
              253
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The pupils stand in a semicircle around the board, the teacher or monitor standing on the left, the head of the class being always on the right.

[1] Amer. Jour. of Educ., 1826, I:35.
[2] Key to Ray's Practical Arithmetic, p. 6.

First method.

Let the first child begin, and say aloud, "6 and 7 are 13." Let the next child say, "and 6 are 19;" and the next, "and 9 are 28;" and the next, "and 7 are 35." The next sets down 5, and if the children are very young, he sets a small 3 under the 5, as a guide to the next, who says, "3 tens carried to 8 tens make 11." Then head begins again, and says, "11 and 8 are 19;" the next says, "and 9 are 28;" the next, "and 9 are 37;" the next, "and 8 are 45;" the next, "and 6 are 51;" the next sets down 1 in the tens place, and puts a 5 under it. The next says, "5 hundreds carried to 6 hundreds make 11 hundreds;" the next says, "and 6 are 17," and so on until the sum is finished.[1]

This "method" is given several variations. Pupils may be called upon "promiscuously." Or each pupil may add a column silently and place the result upon the board. The next is held responsible when the sum is not correct.

These "methods" are simply types of technique for effectively focusing the attention of the class and arousing interest in the work. No fundamental principles of teaching are stated, but these specific rules for carrying on the classroom work are typical of the method of teaching during this period. Objective and examinable results were desired, and devices which would give these, and would secure attention, were accordingly exalted as methods of teaching.

In teaching mental arithmetic a procedure was adopted for the purpose of forcing the continuous attention of the class. As in the case of written arithmetic, the plan was an artificial device. Stoddard gives in his Methods of Teaching the following "methods:"

First method.

The teacher reads the problems and calls upon the different members of the class promiscuously. Each pupil named arises, repeats, and analyzes the problem. Members of the class who have discovered mistakes, or who take exception to the method of analysis, raise their hands, and the teacher designates some one of them to make the necessary correction, or he makes it himself.

Modifications of the above method:

1. Call upon different pupils to solve different parts of the same problem, each as he is named being required to proceed with the analysis where the pupil who has just taken his seat left it. This method furnishes an opportunity for "stirring up," or jogging the memory of the inattentive.

2. The pupil designated to analyze a problem arises, repeats it, and names another to solve it.

Second method.

The teacher reads a problem, the class solves it in silence, and as rapidly as possible; each raises the hand on obtaining the result.

After giving sufficient time the teacher, if he wishes a simultaneous answer, says "Class," and all who can pronounce the result together. Or he names a pupil, who arises, gives the result, and solves the problem.

Third method.

The teacher reads and assigns a problem to each member, or a part of the members, of the class without waiting for a solution. He then calls upon pupils promiscuously

[1] P. 55.

who have had questions given them, and the pupil named arises, repeats, and analyzes the problem assigned him.

This method is good discipline for the memory

Fourth method.

Two pupils are designated as chiefs, and choose alternately from among the other members of the class such as they deem the best scholars in mental arithmetic for a trial of skill. The teacher gives out problems alternately and marks the failures or the number of questions which each side solves correctly, and at the close of the lesson gives the result. Or he causes each pupil that fails to take his seat, and the side that has the largest number of pupils standing at the close of the lesson is pronounced the best.[1]

Motivation.—The plan for securing motive which is offered by the writers on the teaching of arithmetic, whom we have quoted on the preceding pages, is by appeal to artificial incentives. The pupil attended to the example in addition because, if he did not, he knew his failure to attend would be immediately discovered by both his classmates and the teacher. Being thus caught in the act, the penalty followed, a lowering of his rank in class, a reprimand by the teacher, or a severe punishment. Or the pupil wished to secure the approbation of his teacher or parents. Knowing the shortcomings of his classmates he attended in order that he might profit by their failures. As soon as the pupil reciting faltered or made an error he was ready to take up the solution of the problem and receive his reward in the approval of the teacher or in the anticipation of the reception which would be given at home to his report card. Or perhaps his reward came from the superior position which he had attained in the class. In putting one division of the class against the other the instinct of emulation was appealed to. Or where the class was small the contest was between the individual pupils. There was an appeal to the pupil's pride when he knew his work was to be examined and marked by another pupil.

Motive was secured in other ways. The puzzle type of problem stimulated the pupil's curiosity. Some problems were practical. The primary work and the rapid drill were immediately interesting to many. But these ways of securing motive were, for the most part, used unconsciously. When a teacher wrote of how attention was secured these phases of motive were mentioned only incidentally or not at all. Occasionally, but usually in respect to other school subjects, motive by conflict of ideas was mentioned. David P. Page, in a text on *Theory and Practice of Teaching,* gives a list of good incentives. They are: (1) Desire of the approval of parents and teachers; (2) desire of advancement; (3) desire to be useful; (4) desire to do right; (5) natural love in the child for acquisition and a natural desire to know.

[1] P. 25.

The idea and practice of securing motive in this period was characterized by there being no intrinsic relation between the purpose which the pupil recognized and the subject matter studied.

Problems solved according to a formula.—In mental arithmetic, which was considered to be especially suitable for developing the reasoning, the solution was accomplished by applying a syllogistic formula. Ray says in Hints to Teachers, Intellectual Arithmetic (copyright 1860):

A method of solving questions in mental arithmetic now much used is the following, called the "Four-step method:"

Illustrations.—*First step*, James gave 7 cents for apples and 8 cents for peaches; how many cents did he spend? *Second step*, as many as the sum of 7 and 8 cents. *Third step*, 7 cents and 8 cents are 15 cents. *Fourth step*, hence, if James gave 7 cents for apples and 8 cents for peaches he spent 15 cents.

Again: *First step*, 4-fifths of 25 are how many times 6? *Second step*, as many times 6 as 6 is contained times in 4-fifths of 25. *Third step*, 1-fifth of 25 is 5, 4-fifths are 4 times 5, which are 20; 6 in 20 is contained 3 and 2-sixths times. *Fourth step*, therefore, 4-fifths of 25 are 3 and 2-sixths times 6.

Some writers insisted that these forms of analysis were to be committed to memory. In the following quotation the author believes that a verbatim memorizing of the forms of analysis will make the pupils all the better reasoners:

After the pupils are familiar with the process and have received sufficient drill they should be taught to analyze problems. The teacher should see that the analysis is *thoroughly understood and accurately recited*. They should be required to write out an analysis, and the pupil that presents the most *simple* and *concise* analysis should write it on the board, subject to the criticism of the class. See that the language is used correctly; that it tells the "*truth, the whole truth, and nothing but the truth*." Now require every member of the class to commit the analysis verbatim as he would a demonstration in Euclid—for experience teaches that those pupils who are *critically* close in committing verbatim the demonstrations in geometry make by far more *accurate* reasoners and ready mathematicians.[1]

When the pupil was furnished with a stereotyped form for the solution of every problem all opportunity for reasoning was eliminated except such as there might be in identifying the particular problem with the appropriate formula. Therefore the types of problems were mixed, to form promiscuous and miscellaneous lists of problems. Brooks says:

It will be frequently noticed that, after beginning the lesson with the typical problem, variations are made both in the conditions of the question and in their application to other objects than those named in the original problem. This is done to give variety to the exercises and to afford discipline to the pupil.

Assisting the pupil.—The assistance which the teacher rendered the pupil consisted mainly in holding him to certain fixed standards, in drilling upon what was considered fundamental, and in explaining difficult operations and problems. Little effort was made to assist the pupil to think. Developing a process or topic was not consciously

[1] De Graff, The Schoolroom Guide, 1877.

attempted. The explanations by the teacher were simply told to the pupil. Whether the pupil understood the explanation or not, he was expected to remember it. If the difficulty was sufficiently important, the pupil was drilled upon the manner of overcoming it. In this way the learning was largely by conscious imitation, with sufficient repetition by the pupil to fix the subject matter in the mind. It seems to have been recognized that expression assisted in making the impression. The prominence given to the explanations in class by pupils was in part due to this belief.

Deductive method.—We have seen how the texts of this period became deductive in form after 1857. The instruction followed the texts closely. The "rule" was again emphasized. If the pupil was able to subsume a problem under a known rule, the rule could take care of the answer. A report of the investigation of schools in Connecticut, in 1887–88, contains the following comment upon the attitude of the pupil toward the rule:

> The method in arithmetic is illustrated by the course which most children will take after long instruction in such schools. If they are given a problem of one or two steps, *they will first see what rule it comes under.* If it does not come under any rule with which they are familiar, they will take a book and see if they can find an example like it. If they fail in this search, they then begin to cipher at random, multiplying and dividing in the hope that it may turn out right.[1]

The rules were not developed as Colburn did in his texts. The pupil was scarcely allowed to make a hypothesis when a new type of problem was reached, and to work out a solution of his own. Instead the rule was given to him ready made.

Objective teaching.—The use of objective materials, beans, grains of corn, pieces of crayon, etc., is recommended by Ray for the younger pupils. He also describes what he terms "arithmometer," an instrument for representing objectively the number facts of the four operations. However, he cautions against "frequent use of artificial aids," for it "tends to prevent the pupil from exercising his own intellectual powers, and thus, if carried too far, is productive of positive injury." In an edition of Greenleaf's primary book he says:

> The First Lessons in Numbers has been prepared in the belief that the objective presentation of numbers is best suited to the comprehension of the child. The teacher who uses this book is expected to make constant use of counters, blocks, or other visible objects, that from the outset the child may have correct ideas of numbers. The copious illustrations found throughout the book are intended as aids in this direction.

One writer on the teaching of arithmetic (1877) says: "Construct the addition tables at first by the use of objects." He advises the same plan for multiplication. Illustrations in the form of cuts became a feature of the primary arithmetics, but I have found none as profusely illustrated as Emerson's Part First. A few illustrations are found in the "mental" and the "practical" arithmetics, but usually only for elucidating a topic of peculiar difficulty.

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[1] Connecticut School Documents, No. VII, p. 224.

The second Pestalozzian movement in the United States, usually known as the Oswego movement, emphasized almost exclusively objective teaching. This movement, which dates from 1860, appears to have had but little direct influence upon the teaching of arithmetic. There was only a slight increase in the use of objective materials in arithmetic after 1860.

Drill.—We have already shown that skill and thoroughness were emphasized as ends to be attained. They were to be secured by drill. These goals of instruction were given increased importance in the latter part of this period. Drill devices and drill cards were given by a number of authors. Drill to make certain parts of the subject matter mechanical was insisted upon. De Graff, in The Schoolroom Guide, 1877, says:

The teacher should see that the tables are thoroughly committed to memory by requiring pupils to recite them backward and forward regularly and irregularly. Excite emulation among members of the class in regard to the mechanical execution of the work, because careless habits formed will ever be a source of annoyance to both teacher and pupil.[1]

In Felter's Primary Arithmetic the teacher is advised:

In order to secure thoroughness, give short lessons and spend much time in daily review. If in the exercise of "fours," do not proceed until everything that precedes is as familiar as the alphabet. If it required one month, take it; if *one year*, the time can not be better spent. Never allow a pupil who, habitually, misses over 10 per cent of the given exercise to remain in the class.

Some reports upon the teaching of arithmetic.—In 1887–88 a committee reported upon the condition of schools in New London County, Conn. The report was based upon tests and visitation. In respect to arithmetic they say in part:

Perhaps a half-dozen schools taught elementary arithmetic by systematically developing number, but almost invariably the teacher answered the question as to what method was used by the stereotyped phrase "follow the book," and this was literally true not only for elementary but for advanced classes.

Months and terms are spent in counting, learning to write unheard-of numbers, bawling the multiplication and perhaps other tables. No systematic development of number is thought of. No concrete examples, except the few in a small book are given. No thorough drill is attempted. No rapid handling of numbers, no accuracy with figures, no training of the reason, is the result. Most so-called mental examples have been carefully studied before the recitation. Definition and rules will be repeated fluently, and yet the pupil is unable to perform simple examples involving one or two steps of reasoning. One or two illustrations are pertinent.

A boy over 10 years of age was being taught to count one hundred, but could not tell the sum of two and two. The teacher gave as the reason for teaching him thus to count, before he could add, that "when he received change at the store he could count it."

In another school, a class of three gave with great fluency the definition of "units," "arithmetic," "counting," "scale," "counting off," "group," etc. They read numbers up to sextillions, but could not tell how many fours there were in 16. The teacher said that they had never done anything in multiplication or division. These

[1] P. 189.

children had been in school about four years. It is not to be wondered at, then, that under such unnatural methods so many children attend school seven and eight years without reaching percentage and its applications to interest.[1]

A more elaborate investigation was made of the schools of New Haven County, Conn., in 1890–91. The committee examined 167 districts. They sum up their opinions with respect to arithmetic in this sentence:

Arithmetic has thus become a science of difficult trifles and intricate fooleries peculiar to common schools, and remarkable chiefly for sterility and ill-adaptedness to any useful purpose.

In another place they give a description of the actual activities of the schoolroom which is illuminating not only as to content, but to aim and method as well:

In many districts the main thing in arithmetic is the definitions. In one school the first class was questioned as follows: "Spell arithmetic." "What is arithmetic?" "What is Roman notation?" "What is a figure?" and so on during the recitation periods. The definitions must be word for word as in the book.

The answer to the question, "How is a fraction expressed?" was given "by writing one number above the other." This was immediately corrected by the teacher to "by placing one number above the other."[2]

While these surveys were made with some care, they covered only a very limited area. For this reason it is hazardous to draw generalizations, but other evidence indicates that the conditions described in these reports are typical of much of the instruction in arithmetic at this time. However, when we compare the instruction in arithmetic during this period with that of the ciphering-book period, progress is shown. There is an enlarged concept of the function of the teacher, and if we take into account that at this time arithmetic was a "required" subject and not an "elective," it is probable that the results secured were superior.

Colburn's influence upon the teaching of arithmetic.—A comparison of Colburn's method of teaching arithmetic with the practice during this period reveals that he influenced the teaching of the subject much less than he did the texts. The use of objective materials and the oral instruction which mental arithmetic made necessary may be attributed to him.

He advocated class instruction and discussed its technique in his address on the "Teaching of arithmetic," but it is doubtful if the adoption of this plan of instruction was due to his advocacy of it. Outside of these features, which were a result of his texts rather than his presentation of the method of teaching arithmetic, there is little trace of Colburn's influence. This condition is easily explained by the fact that a method of teaching is much less tangible than the form of a textbook. Also, in the making of texts there are few persons concerned, as compared with the number of teachers.

[1] Connecticut School Documents, No. VII, p. 117. [2] Idem, No. XVI, p. 29.

PART IV.

Chapter X.

RECENT TENDENCIES AND DEVELOPMENTS.

The development of arithmetic since 1892 has been more intimately connected with the general educational development than it was during the period beginning with 1821. An attack upon the importance of the disciplinary function of arithmetic grew out of two more general movements.

The Herbartian movement.—Beginning about 1890, American educators were greatly interested in the educational principles enunciated by Herbart, a German educator who lived from 1776–1841. Some of the important events in the rise of this movement were: The publication of educational books on Herbartian principles; Essentials of Method, by Charles De Garmo, 1889; General Method, by Charles McMurry, 1892; The Method of the Recitation, by Frank McMurry and Charles McMurry, 1897; and the formation of a national Herbartian society in 1892. The principle of apperception, which is one of the most important accredited to Herbart, was emphasized by his followers in America. Briefly the principle is this:[1] New experiences are given meaning and interpreted by means of the ideas which one has obtained from his past experience and which are present in his consciousness at the time. This principle, coupled with Herbart's concept of the immediate end of education as the development of a "many-sided interest," means that education is to give the child (1) a "many-side" acquaintance with the external world, and (2) to give this acquaintance in such a way that it will be accompanied by an active "interest" in each "side" of this experience. The child will then be equipped to meet new situations as they arise.

This theory, which places the emphasis upon the content of a subject, is fundamentally opposed to the disciplinary concept of education, and the wave of enthusiastic interest in the work of Herbart which swept over the United States did much to counteract the great emphasis upon the disciplinary function of instruction in arithmetic. The Herbartians emphasized history and literature as subjects in the elementary school, and by so doing were a factor in reducing the amount of time given to arithmetic.

[1] For Herbart's own account, see Outlines of Educational Doctrines.

The psychological movement.—In his "Principles of Psychology," 1890, William James stated that one's native ability to retain can not be changed,[1] which means that a general capacity to remember can not be trained by specific exercises. This assertion, which was "supported by some plausible experimental evidence" was extended by other educators to a complete refutation of the theory of formal discipline as then interpreted.[2]

The reaction against the disciplinary value of arithmetic.—Coupled with these two movements, partly as a result of them, both educators and the public became more actively critical of the work of the public schools.[3] There were reports of investigations[4] and more general utterances based upon general observations. The following are typical of this latter type:

> In almost all of the arithmetics, first come the definitions, then the rules, then a problem with full explanation, then the problems for the children to work according to rule and like the sample given. And this is called discipline! God save the mark![5]
>
> From one-sixth to one-fourth, or even one-third, of the whole school time of American children is given to the subject of arithmetic—a subject which does not train a single one of the four faculties that it should be the fundamental object of education to develop. It has nothing to do with observing correctly, or with recording accurately the results of observation, or with collecting facts and drawing just inferences therefrom, or with expressing clearly and forcibly logical thought.[6]

In 1892 the Committee of Ten, a committee appointed by the National Education Association, recognized the existence of a formal disciplinary value of arithmetic,[7] but insisted that it was "greatly inferior to what may be obtained by a different class of exercises." Essentially this is a refutation of the doctrine of formal discipline as it had been applied to arithmetic.

Simon Newcomb, who was chairman of the subcommittee on mathematics, stated in another place that "the main end of mathematical teaching—we might say of teaching generally—is to store the mind with clear conceptions of things and their relations."[8]

Charles A. McMurry stated that the "chief aim of arithmetic is the mastery of the world on the quantitive side through number concepts."[9]

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[1] Vol. I, p. 667.

[2] See W. C. Bagley: Educational Values, p. 188 ff., for a historical account of the reaction against formal discipline.

[3] See the proceedings of the National Educational Association following 1880 for criticisms by the leading educators of the country. These have particular reference to arithmetic, 1880, p. 114; 1881, pp. 24, 85, 106; 1892, pp. 617, 620.

[4] See pp. 126-27 for the report of investigations of the teaching of arithmetic in Connecticut.

[5] O. F. Bright, "Changes in Schools," Proc. of Nat. Educ. Assoc., 1895, p. 263.

[6] Statement by Charles W. Eliot, 1892. Quoted by O. F. Bright, "Changes in Schools," Proc. of Nat. Educ. Assoc., 1895, p. 264. See also, Charles W. Eliot: Educational Reform, pp. 186-187, Address, 1890.

[7] Report of Committee of Ten, p. 106.

[8] Simon Newcomb: "Methods of Teaching Arithmetic," Educ. Rev., vol. 31, pp. 339-340. This point of view was originally expressed in 1892.

[9] Special Method in Arithmetic, 1895, p. 16.

These last two statements are representative of the pure Herbartian point of view and of the extreme reaction against the importance of the disciplinary function of arithmetic.[1]

Arithmetic a psychical and social demand.—The most important constructive contribution of this period was made by Prof. John Dewey,[2] whose fundamental thesis was that the psychical and social environment in which we live presents problems which the human mind solves by measurement, i. e., by number and number relations.[3] Thus number is not a property of objects, but rather it is "the product of the way in which the mind deals with objects in the operation of making a vague whole definite."[4] The necessity for making these vague wholes definite grows out of the fact that (1) material things are "limited," (2) that energy must be economized, and (3) that remote ends must be attained.

Dewey illustrates these reasons as follows:

(1) If every human being could use at his pleasure all the land he wanted, it is probable that no one would ever measure land with mathematical exactness. There might be, of course—Crusoe like—a crude estimate of the quantity required for a given purpose; but there would be no definite numerical valuation in acres, rods, yards, feet. There would be no need for such accuracy. If food could be had without trouble or care, and in sufficiency for everybody, we should never put our berries in quart measures, count off eggs and oranges by the dozen, and weigh out flour by the pound.

(2) Because there is a limit to human energy, when we employ this energy for the attainment of a purpose, the most fruitful results are attained when there is the most accurate balancing of the energy over against the thing to be done. If the arrow of the savage is too heavy for his bow, or if it is too light to pierce the skin of the deer, there is in both cases a waste of energy. If the bow is so thick and clumsy that all his strength is required to bend it, or so slight or uneven that too little momentum is given to the arrow, there is but a barren show of action, and the savage has his labor for his pains. Bow and arrow must be accurately adjusted to each other in size, form, and weight; and both have to be equated (as the mathematician would say) or balanced to the end in view—the killing of the game.

(3) In working out a certain purpose, for example, one of a series of means is a journey to be undertaken; it is of a certain length, it is to be completed in a given time and within a certain maximum of expense, etc., and this involves careful calculation, measurement, and numerical ideas.

To this principle Dewey added his more general educational principle that the process of education is most efficiently carried on when the child is placed in the physical and social environment which demand psychical activity.[5] Applied to arithmetic, this means that to teach it efficiently the school must produce situations which call for measurement and the relating of quantities. According to this thesis the immediate purpose of the author of a text and of the teacher

[1] For the other extreme see p. 91.
[2] James A. McLellan and John Dewey: The Psychology of Number, 1895.
[3] Op. cit., pp. 24, 32, 35 ff.
[4] Op. cit., p. 32.
[5] Op. cit., p. 23.

would be to provide these situations. When taught from this point of view, arithmetic affords "an unrivaled means of mental discipline," but Dewey does not use "mental discipline" in the sense in which it was used by the authors of arithmetics during the previous period.

Both the disciplinary and the utilitarian functions of arithmetic recognized at present.—Through the publication of The Psychology of Number, and through the exemplification of his principles in the University Elementary School, Dewey combined with the reaction against the doctrine of formal discipline in influencing the concept of the aim of instruction in arithmetic. Soon after the publication of The Psychology of Number two series of arithmetics were published which the authors claimed embodied the principles enunciated by Dewey.[1] Other authors builded upon them in part. These texts have been a factor in increasing the emphasis upon problems taken from practical situations, and hence upon the utilitarian value of arithmetic.

The reaction against formal discipline was followed by a counter action in which educators have recognized the disciplinary function of arithmetic,[2] but in general they accorded the utilitarian value equal rank, and this appears to be the present status. A recent questionnaire was sent to 185 State normal schools and to 8 city training schools. Replies were received from 65 State normal schools and 3 city training schools. In training teachers of mathematics

> 51 per cent of these schools claim to pay equal attention to mathematics as a science (the so-called culture value) and to mathematics as an art (the so-called utilitarian value). About 28 per cent claim to emphasize more the cultural aspect (except in arithmetic), and 21 per cent put greater stress upon the utilities.[3]

In all of this agitation there seems to have been the underlying purpose to adapt arithmetic to the nature of the child and to the social demands which will be made upon him when he leaves school.

Definition of the aim of instruction.—Recently scientific investigation has revealed that the product of instruction in arithmetic is not a single ability but consists of many abilities.[4] The ability "to add columns three figures long is not the same ability as to add columns five figures long." Each type of example calls for a different ability, and thus the product of instruction in arithmetic includes as many different abilities as there are different types of examples. Courtis has identified 15 different addition abilities,[5] 8 for subtraction, 11 for multiplication, and 14 for division.

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[1] One series was by McLellan and Ames, 1898; the other by Belfield and Brooks, 1898.

[2] See J. W. A. Young: The Teaching of Mathematics, 1906, p. 204; and D. E. Smith: The Teaching of Arithmetic, 1909, p. 11ff.

[3] Training of Teachers of Elementary and Secondary Mathematics, U. S. Bu. of Ed. Bul., 1911, No. 12, p. 15.

[4] C. W. Stone: Arithmetical Abilities and Some Factors Determining Them, 1908, pp. 42–43. S. A. Courtis: Teachers Manual, 1914, p. 2.

[5] Op. cit., p. 2.

This analysis of the product of instruction in arithmetic has made possible more exact and objective definitions of the aim of instruction. At present this has been done by Courtis for the fundamental operations with integers.[1] For any particular grade the teacher and pupils have for their aim, in so far as it involves the fundamental operations with integers, to attain the ability to solve examples of certain types with a specified speed and accuracy. These standards are based upon extensive experimental data gathered from both schools and the commercial world. These detailed and objective statements of aims of the instruction in arithmetic are not opposed to, but will supplement, the more general statement of aim.

Less time given to arithmetic in the schools.—During the preceding period a large per cent of the total school time was given to arithmetic. Estimates ranged as high as 50 per cent or more.[2] The Committee of Ten, 1892, stated that a "radical change in the teaching of arithmetic was necessary," and recommended that the course be both "abridged and enriched." In 1895 the Committee of Fifteen reported as follows:

> Your committee believes that, with the right methods and a wise use of time in preparing the arithmetic lesson in and out of school, five years are sufficient for the study of mere arithmetic—the five years beginning with the second school year and ending with the close of the sixth year; and that the seventh and eighth years should be given to the algebraic method of dealing with those problems that involve difficulties in the transformation of quantitative indirect functions into numerical or direct quantitative data.
>
> Your committee is of the opinion that the so-called mental arithmetic should be made to alternate with written arithmetic for two years, and that there should not be two lessons daily in this subject (arithmetic).[3]

In another place the committee reports "that the practice of teaching two lessons daily in arithmetic, one styled 'mental' or 'intellectual' and the other 'written' arithmetic, is still continued in many schools."[4] Although there was a marked tendency even before 1892 to combine "mental" and "written" arithmetic in the texts, this practice persisted in some places until very recently. Separate classes in mental arithmetic were discontinued in Kansas City, Mo., in 1913.

The following data show the change in the relative amount of time given to arithmetic in several American cities.

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[1] Op. cit., p. 3.
[2] See page 126.
[3] Proc. Nat. Ed. Assoc., 1895, pp. 300-301.
[4] Ibid., p. 299.

Per cent of total school time given to arithmetic.[1]

Cities.	1888	1890	1904	1910–11
Boston		16.6	16.2	15.5
Chicago		9.3	18.8	10.0
Cleveland		14.1	17.3	15.5
Columbus, Ga			18.6	
Jersey City			19.5	
Kansas City			15.1	
Louisville			17.2	
New Orleans			18.6	
New York	26.2	26.2	12.0	13.4
San Francisco		14.0	15.3	16.6
St. Louis	19.3	19.3	15.2	15.0
Baltimore		19.5		18.3
Cincinnati		13.4		18.8

[1] The data for 1888 and 1904 are taken from B. R. Payne: Public Elementary School Curricula; for 1890 and 1910–11 from U. S. Bureau of Education, Bulletin No. 4, 1911. For 1890 and 1910–11, algebra taught in the grades is included.

A recent investigation[1] of 50 of the leading American cities shows that 15.26 per cent of all of the school time is devoted to arithmetic. However, in the report of the same investigation it is stated that mathematics is "less prominent in city systems than in the rural districts." This indicates that 15.26 per cent is too low for the country as a whole.

While the change in the relative time allotment has been irregular in many of the cities, and there is little uniformity, the tendency in city schools seems to be to give a little more than 15 per cent of the school time to instruction in arithmetic, which probably represents a decrease of 10 to 25 per cent since the middle of the past century.

The content of the texts.—Arithmetics are usually published in the form of a series which consists of a primary text followed by one or two books for the upper grades. With very few exceptions the primary texts are intended to be completed by the end of the fourth grade.

In these primary texts, pictures and graphical designs are employed for representing objectively numbers and number relations. It is frequently suggested that the teachers introduce objects for this purpose. Denominate numbers are introduced very early in the texts. Some authors intend that the relations between quantities, such as pint, quart, and gallon, shall be developed by the children, and most authors intend that the children shall have some first-hand acquaintance with the most common measures.

Dewey contended that number and number relations were the product of measuring.[2] Soon after the appearance of The Psychology of Number, in 1895, measuring was made a prominent feature of a few series of arithmetics, especially the primary texts. In more recent texts it has been given a place, though with varying degrees of emphasis. Pupils are asked to tell which is the longest

[1] U. S. Bu. of Educ., Bul., 1911, No. 13.

[2] See p. 130.

or which the shortest of a group of lines, to estimate the length of lines, to fold paper figures of given dimensions, etc. Tables of denominate numbers are developed by measuring. In the more advanced grades, the pupils gather data for some of their problems by measuring city lots, school gardens, and from the measuring in making articles in the manual training shop and in cooking in the domestic science laboratory. Counting, as a form of measuring, is very conspicuous in some texts. The counting of objects was a part of the plan of Colburn. But now they count by twos, by threes, by fours, etc., not counting objects, just counting.

In some texts there is an attempt to have the child engage in activities which will demand a knowledge of number and number relations. Number games, such as ring-toss, bean bag,[1] etc., are varied in such ways that fundamental number facts are demanded in determining the relative standing of the participants. Keeping store, cooking, and other construction work are used to create situations which require a knowledge of number and number relations. More frequently the author of the text describes a game, a store, a bank, a farm, a factory, or some other activity, and then gives a series of problems of the type which do arise in such situations. It is suggested that when possible the pupils be taken to visit the actual industry. An understanding of the activity in which the problem occurs is regarded as a legitimate phase of arithmetic.

The problems in these lists are for the most part such as actually do arise in the given activity, and there is an increasing tendency to go to the occupation and take problems which have actually arisen. But as yet this has not been done very consistently. There has been a very pronounced tendency to use excessively large numbers in the problems. For example, in a popular text the first problem under the head, "Our Forests," is: "If there are 672,000,000 acres of woodland in the United States, how many square miles are there?" Each problem in the list involves a number as large as a million. Besides the practical problems which are thus grouped in lists, there are many drawn from a wide range of sources. This range suggests the plan of Horace Mann,[2] but he probably influenced the present situation only slightly, if at all. Problems of the type, "The area of the Atlantic Ocean is 24,651,410 square miles and this is 49 per cent of the area of the Pacific Ocean. What is the area of the latter?" are prevalent.

The Speer method.—Another interpretation of the child with respect to arithmetic was made by William W. Speer. The basis of his plan for arithmetic was that number was a ratio obtained by comparing two magnitudes. He devised a set of solids which the

[1] A number of such games are described in The Teaching of Arithmetic, by D. E. Smith, ch. 14.
[2] See p. 117.

pupil was to handle and compare. The pupil's idea of number and the operations upon them were to come from these activities. This idea of ratio permeates the whole of his texts, which were published in the later nineties. The plan had been conceived by Tillich many years before. However, there is no direct evidence in Speer's texts to show that he was indebted to Tillich. The question of the originality of the work is of little importance, for, while the "Speer method" attained some popularity, it never became widely used, and there is very little trace of it in our present popular texts.

Omissions.—The texts of the previous period contained topics and problems which had little or no practical value. The report of the Committee of Ten, which we have taken as marking the beginning of the recent period, contained the following recommendations with reference to these topics:

> Among the subjects which should be curtailed, or entirely omitted, are compound proportion, cube root, abstract mensuration, obsolete denominate quantities, and the greater part of commercial arithmetic. Percentage should be rigidly reduced to the needs of actual life. In such subjects as profit and loss, bank discount, and simple and compound interest, examples not easily made intelligible to the pupil should be omitted. Such complications as result from fractional periods of time in compound interest are useless and undesirable. (P. 105.)

F. M. McMurry, in an address before the National Department of Superintendence, 1904, enumerated a list of topics which he thought might well be omitted. The curtailment which he advocates is somewhat in excess of that recommended by the Committee of Ten, but in general agrees with it.

In 1911 the International Commission on the Teaching of Mathematics, referring to the above recommendations, report that "only 38 per cent of the 50 largest cities have followed out the recommendation."[1]

In a more recent investigation[2] a questionnaire was sent to city superintendents to which 867 replies were received. A majority of these favored eliminating apothecaries' weight, furlong, dram, quarter in avoirdupois, compound proportion, unreal fractions, alligation, and progression, and less than a majority favored eliminating Troy weight, rood in square measure, surveyors' tables, foreign money, folding paper, reduction of more than two steps, long method of greatest common divisor, least common multiple, true discount, cube root, partnership, compound and complex fractions, cases in percentage, annual interest, longitude and time, metric system, and aliquot parts. However, a majority favor either eliminating or giving less attention to all these topics. On the other hand, from three-fifths to three-fourths of these superintendents favored giving more attention to addition, subtraction, multiplication, division, and

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[1] Mathematics in the Elementary Schools of the United States, U. S. Bu. of Educ. Bul., No. 13, 1911, p. 17.
[2] W. A. Jessup: Economy of Time in Arithmetic, Ele. Sch. Teach, Vol. XIV, p. 461.

fractions. A majority favored giving more attention to saving and loaning money, taxes, public expenditures, insurance, and public utilities.

The course of study.—The grade occurrence of arithmetic topics based upon 47 courses of study is given in the following table:[1]

Arithmetic topics in the various grades. Connersville (Ind.) public schools.

Subjects.	Grades.							
	I	II	III	IV	V	VI	VII	VIII
Numeration	37	37	33	18	7	4	4	1
Notation	34	39	33	18	7	4	4	1
Relation of numbers	10	11	6	5	5	4	4	3
Addition	23	39	33	28	13	9	6	3
Subtraction	24	39	34	26	13	9	6	3
Multiplication	10	28	30	25	15	15	10	10
Division	2	16	21	34	27	25	19	17
Fractions	10	28	22	31	34	28	24	24
Denominate numbers	11	20	23	34	30	31	29	22
Involution and evolution					1	1	7	16
Decimal fractions				5	23	12	8	1
Mensuration	6	8	10	11	12	14	14	12
Multiplication tables	2	8	20	18	7	5	7	6
Commission and brokerage						10	11	6
Insurance						10	9	6
Percentage					7	16	13	8
Ratio and proportion				1	3	8	6	9
Partnership						2	7	4
Partial payments							3	5
G. C. D. and L. C. M					4	6		
Longitude and time						4	7	1
Profit and loss						7	17	2
Taxes						2	14	3
Duties						1	13	1
Banking							9	6
Exchange							4	4
Simple interest				1	2	12	23	4
Stocks and bonds							6	8
Business forms					1	4	15	6
Simple accounts				3	6	5	3	3

Since only 47 courses are included, it is very evident that several topics must be taught in one or more grades; and, on the other hand, some of the topics have been eliminated in some of the cities. For example, partial payments occur only 8 times in 47 courses of study. Hence this topic has been eliminated in at least 39 out of the 47 cities.

The International Commission on the Teaching of Mathematics summarized their findings on the course of study as follows:

Grade 1.—More or less incidental number work or number work correlated with manual training or with some other definite subject. Variations: From no number work at all to very formal work on addition, subtraction, and the multiplication tables.

Grade 2.—Number work correlated with other subjects. Addition facts emphasized and in many places the multiplication table begun. Variations: In a few schools there is no number work; in some, at the other extreme, division is taught.

Grade 3.—The process of addition and subtraction mastered, together with some work on the multiplication tables, the tables often being completed. Variations: A few schools give no work at all, while some give considerable work in fractions.

[1] This table is from the Course of Study in Mathematics, Connersville (Ind.) Public Schools, 1911. The tabulation was made by Mr. G. M. Wilson, then the superintendent of schools. The courses of study represented cities in 33 different States.

Grade 4.—Multiplication and division mastered. Variations: Fractions are taken up in many schools.

Grade 5.—Fractions mastered, some decimals introduced, denominate numbers employed.

Grade 6.—Decimals as related to common fractions, with much problem work. In some schools simple interest and percentage are begun.

Grade 7.—Percentage and some of its applications.

Grade 8.—Business applications of percentage; mensuration of solids. Variations: No arithmetic at all in the whole or latter half of the grade; the time devoted to algebra; algebra combined with arithmetic.[1]

Returns from 754 cities show that seven-tenths of 1 per cent introduce a text in the first grade, 8.7 per cent in the second, 56.1 per cent in the third, 27.7 per cent in the fourth, 6.1 per cent in the fifth, and seven-tenths of 1 per cent in the sixth.[2]

The variations in school practice which these investigations show are significant. The courses of study have not been constructed scientifically. The occurrence of topics is due to tradition and opinion. Many of the distributions of the occurrence of topics resemble chance distributions.[3]

During the period from 1821 to 1892 the systematic study of arithmetic was usually begun when the child first started to school. Sometimes that was as early as the age of 3 or 4.[4] The Committee of Ten say: "The course in arithmetic thus mapped out should begin about the age of 6." F. M. McMurry said, in 1904:

> In addition to all of these, arithmetic may be omitted as a separate study throughout the first year of school, on the ground that there is no need of it, if the number incidentally called for in other work is properly attended to.

Some writers within the last few years have gone on record as saying that the systematic study of arithmetic should begin in the fourth grade. Investigations show that in some cities no systematic arithmetic is taught in the first three grades, and it appears that there has been a movement in the direction of delaying the systematic study of arithmetic until about the fourth grade. When arithmetic is not studied systematically in these primary grades, incidental instruction in the subject is usually given by means of number games and in connection with the other subjects.

The organization of the texts.—Prior to this period the general plan of the practical arithmetics had been topical, i. e., addition was completed before subtraction was begun, and in turn subtraction was completed before multiplication was begun. This was also partially true for the primary books, but beginning about 1896 texts began

[1] Op. cit., pp. 18-19.

[2] W. A. Jessup: Grade for Introduction of a Text in Arithmetic. El. Sch. Jour., Vol. XV, p. 163 (Nov., 1914).

[3] See Mathematics in the Elementary Schools of the United States. U. S. Bu. of Educ., Bul. No. 13, 1911, pp. 19-32.

[4] See p. 92.

to appear which were organized upon what has been known as the "spiral plan."

The Werner Arithmetics, by Frank H. Hall a three-book series for graded schools, and *The Hall Arithmetics* by the same author, a two-book series for graded or ungraded schools, were pioneers in the exploitation of the spiral plan of organization. A copy of the Werner Arithmetics, Book I, which I have bears the date 1896. As to the plan of this text, Mr. Hall says in the preface:

The first five lines of this book present problems in addition, subtraction, multiplication, division noting the number of groups, and division noting the number in each group. Then, by a kind of spiral advancement, the pupils move around this circle and upward through all the intricacies of combination, separation, and comparison of numbers.

The arrangement of topics is unique and convenient. In this book measurement problems appear on pages 43, 53, 63, 73, etc.; a certain class of fraction problems on pages 45, 55, 65, 75, etc.; facts of addition, subtraction, multiplication, and division on pages 41, 51, 61, 71, etc. This decimal arrangement of subjects makes the books almost as convenient for reference as are the books that are made on the strict classification plan, while the frequent recurrence of similar matter insures thorough review.

This spiral arrangement, which was followed in the other texts of the series, found favor very quickly. Within the next few years a number of texts were published which were organized upon the spiral plan. A few were as extreme as The Hall Arithmetics, but in general the spiral plan was modified in part. The spirals were less numerous, and the "decimal arrangement" was not followed. Within the last few years there has been a pronounced reaction. The spiral plan has been severely criticized, and authors of some of the spiral texts have found it necessary to revise them, eliminating some of the spirals. At present the consensus of opinions seems to be in favor of a moderate spiral for grades one to four, a topical plan for grades seven and eight, and a transition from the one plan to the other in grades five and six. Some authorities, while agreeing with the above general opinion so far as the actual instruction is concerned, contend that the best results can be obtained from using a topical text above the primary grades. The teacher can then adopt a spiral which will meet more nearly the needs of the community and the particular class.

The Grube method.—Grube (1816–1884) was a German whose "claim to rank as an educator lies largely in his power of judicious selection from the writings of others."[1] The features of Grube's writing which stand out most clearly are objective teaching, the measuring of each number with fixed units, the spiral or concentric circle plan of organization, thoroughness and complete mastery, making of each arithmetic lesson a language drill, and the simultaneous teaching of the four fundamental operations for each number.

[1] D. E. Smith: The Teaching of Elementary Mathematics, p. 80.

Grube presented his method in "Leitfaden für das Rechnen in der Elementarschule, nach den Grundsätzen einer heuristischen Methode" (Guide for Reckoning in the Elementary School, according to the Principles of a Heuristic Method). This was published in 1842. The beginning of the method in this country dates from 1870, when F. Louis Soldan presented to the teachers' association of St. Louis an account of Grube's plan for teaching the numbers 1 to 10. The plan was tried in the St. Louis schools and later elsewhere. In 1876 Soldan presented the remainder of Grube's plan, which includes the numbers 10 to 100 and above, and common fractions. This was intended to cover the work of the first four years. In 1888 Levi Seeley wrote *Grube's Method of Teaching Arithmetic*. This is really a complete text for the first four years.

The method rapidly became popular in many sections of the country. One writer suggests that the reason for the popularity of the method in this country was due to Grube's original treatise being brief and written so as to be easily translated and to the fact that it was a "German" method. Furthermore, it seems that the friends of the method, or at least those who first used it, saw most clearly the good features and emphasized them to the partial or entire exclusion of the less desirable features. Doubtless they, in their enthusiasm, secured commendable results. But as is often the case, as the method was passed on to other teachers, the attention was fixed primarily on the most obvious phase of the method, which said that the four fundamental operations should be taught for each number before the next was taken up. Thus within recent years this single feature has come to stand for the Grube method.

Grube's method has been severely criticized by several recent writers on the teaching of arithmetic. As a plan of teaching it has been discredited.[1] Much of Grube's method was not new to the United States. In fact all the features are to be found in texts published prior to 1870, though some were not given quite as extreme form. Objective teaching began with Colburn. Davies held that the unit was the basis of all numbers and treated each number "as a collection of units." Emphasis had already been placed upon thoroughness and drill in language. In the Child's Book of Arithmetic, 1859, D. P. Colburn approximates the concentric circle plan and the simultaneous teaching of the four fundamental operations. However, this does not alter the fact that Soldan introduced the Grube method directly from the writings of Grube.

The relation of the Grube method to the spiral plan.—The opinion is prevalent that the spiral plan is simply an outgrowth of the Grube method. However, the writer has failed to find evidence to show

[1] See D. E. Smith: The Teaching of Elementary Mathematics; McMurry: Special Method in Arithmetic; McLellan and Dewey: Psychology of Number. It is also true that, in its entirety, the method did not gain any considerable prestige.

this. In fact, there is evidence to indicate that the spiral plan was the result of attempting to fit the organization of arithmetic to the child and to secure thoroughness.

The texts of the previous period were topical, but the order pursued by the pupil was spiral. Not only this, but there were frequent review exercises. Now, when the slogan was "adapt arithmetic to the child," what would be more natural than to put the spiral into the text rather than leave it to the pleasure of the teacher. Thoroughness was the cry, and psychologists were saying that only by repetition is thoroughness secured. Then, the plan of the text make repetition certain. Frank H. Hall suggests this conclusion when he says, "Proper sequence with reference to the pupil has been constantly in the thought of the author in his selection and arrangement of matter," and later, "the frequent recurrence of similar matter insures thorough review."

A careful comparison of the Grube method and the spiral plan reveals many essential differences and few points of contact. Grube did not go beyond the work of the fourth grade. Within the year his spirals were all of the same size; no new matter was admitted in the successive revolutions. The spiral plan usually provided for some preliminary number work in which the pupils learned to count (Hall says up to 100). Also they learned some of the number facts. Grube did not provide for this. The spiral plan did not make the magnitude of the numbers the basis of the spirals. Furthermore, the work of Grube had been severely criticized by Dewey in 1895.

Rationalizing the teaching of arithmetic.—The changes in the aim, subject matter, and organization of arithmetic, together with other factors, have combined to change the method of teaching arithmetic. The present period has been one of transition. Perhaps less has been accomplished in modifying the method of teaching than in the other aspects of arithmetic. Certain it is that school practice has fallen far short of realizing the ideals of method proposed by leaders in arithmetical reform.

The most important factor in this transition has been the child, and progress has been made in the direction of adapting the method of teaching to the nature of the child as revealed by modern psychology. But this progress has been attended by unfortunate wanderings after "single idea methods" and devices. However, the period has been marked by progress. Methods which in themselves are open to serious criticism have rendered service by making obvious defects of the dogmatic, memoriter, disciplinary methods of the past.

Development of topics.—It follows immediately from Dewey's first thesis that the pupil's understanding of number and operations with numbers must result from his own psychical activity. This implies that it is the function of the teacher to provide situations which will exercise the pupil's mind and to simply guide the pupil in this activity. The method of such teaching would consist of a plan for providing situations which call for the use of number and number relations, for moving the pupil to work upon them, and for guiding the activity of the pupil. The plan for guiding is to be based upon the normal way in which the child's mind works in "making a vague whole definite."

The Herbartian plan.—The leaders in the Herbartian movement in the United States emphasized inductive thinking, and their concept of inductive teaching became quite popular and was applied to arithmetic along with other school subjects.

Charles A. McMurry says:

"The study of arithmetical processes furnishes one of the best opportunities to apply inductive methods. And nearly every topic in arithmetic has these two phases: First, to derive these general processes, second, to apply them variously to important practical and theoretic affairs that need arithmetical clarification.[1]

The derivation of "these general processes" consisted of the steps (1) preparation, (2) presentation, (3) comparison and abstraction, and (4) generalization.

The Herbartian plan applied to arithmetic was received with enthusiasm by many teachers and was by them unquestioned. The attempts to use it in actual teaching became and are to-day widespread. But the Herbartian plan of inductive development has been severely criticised,[2] and it has been pointed out that it is a special case of reflective thought with the steps of problem, data, hypothesis, and verification. In addition, the practice of developing or rationalizing every topic in arithmetic has been criticized recently by some educators. They point out that some parts of arithmetic, such as the fundamental operations, must be reduced to habit if they are to function efficiently. They contend that to attempt to explain the "why" in such processes as "carrying" in addition and "borrowing" in subtraction "is merely to stir up unnecessary trouble, trouble unprompted by any demands of actual efficiency."

This position with respect to rationalization is summarized by Suzzalo in his book, The Teaching of Primary Arithmetic, 1912.

He says:

(1) Any fact or process which always recurs in an identical manner, and occurs with sufficient frequency to be remembered, ought not to be "rationalized" for the pupil, but "habituated." * * * (2) If a process does recur in the same manner, but is so little used in after life that any formal method of solution would be forgotten, then the teacher should "rationalize" it. * * * (3) If the process always does occur in the same manner, but with the frequency of its recurrence in doubt, the teacher should both "habituate" and "rationalize." * * * (4) When a process or relation is likely to be expressed in a variable form, then the child must be taught to think through the relations involved, and should not be permitted to treat it mechanically, through a mere act of habit or memory.

[1] Special Method in Arithmetic, p. 60. See pp. 165–167 for some lessons planned according to this method.
[2] See S. C. Parker: The History of Modern Elementary Education, p. 425 ff. for a summary of these criticisms.

This reactionary movement should not be interpreted to mean a return to the former memoriter plan of instruction and drill. It indicates rather that mental processes are being carefully examined and modes of instruction are chosen with reference to the subject matter involved and the end sought.

In the Herbartian plan, deduction came in the step of application and was treated as an incident in the total cycle of inductive development. But it has been pointed out that in life we make many deductions for every induction, and that in the rationalizing of arithmetic deduction has an important place. It is also a special form of reflective thinking, the distinction being that the general principle is a part of the data, and the hypothesis consists in subsuming the particular case under the appropriate rule.[1]

Motivation. Interest as a motive.—Along with the efforts to adapt the mode of instruction to the child, there have been endeavors to work out plans for securing incentives for the mental activity of the child. In reacting from the plan of securing motive by rivalry, emulation, fear of punishment, etc., interest was conceived of as a motive, and the plans for securing motive were plans for arousing interest. Interest and its attendant conditions were very imperfectly understood by the great majority of teachers, and blunders were made in attempts to arouse interest.

For instance, it was proposed that children like easy things, that difficulties were uninteresting. Hence to make arithmetic interesting, make it easy. So difficulties were divided and subdivided or removed. The pupil was "prepared" by the teacher for each topic. And this anemic subject matter was to be interesting and attractive to the pupil because it was easy. Or, the uninteresting became interesting when "associated" with the interesting. Hence to make an uninteresting topic in arithmetic interesting, "associate" it with some activity which the pupil has already found interesting. For example, children are interested in games; they are not interested in the multiplication table. Thus to secure interest in the multiplication table, associate it with some game. This has been done by devising a game with which the multiplication facts could be "associated."

The efficacy of this plan depends upon the interpretation of the word "associate." If it is taken to mean that the game is to be so arranged that the pupils will need, or find useful, the number facts to be taught, the number facts will become interesting. They are then a means to a valuable end. But this was not the way it sometimes worked out in practice. As recently as 1911, an author gives a lesson plan of teaching the multiplication table of fours in which the game of bean bag is to be utilized. The pupils have already played it in

[1] For deduction applied to arithmetic, see Strayer: A Brief Course in the Teaching Process, p. 182.

learning the tables of twos and threes. According to this plan the teacher says to the pupils: "If we are to make large scores, what table must we learn next? How many think they can learn half of the table of fours to-day? If you learn it, we will play our game ten minutes."[1] In this case the "association" of the game with half the table of fours consists of holding up the game and its attendant pleasure as a bribe for memorizing three multiplication facts. The subject matter bears no intrinsic relation to the value to be controlled.

In contrast to this emphasis upon making arithmetic interesting, there is an increasing tendency to recognize that parts of arithmetic, perhaps most of arithmetic, are in themselves immediately interesting to children. This is particularly true of the work of the primary and intermediate grades. D. E. Smith says: "Such statistical information as we have shows that arithmetic has always been looked upon by children as one of the most interesting subjects of the course."

Since the publication of The Psychology of Number there has been an increasing tendency to secure motive for work upon arithmetic by having the child feel the need for number and number relations before he is asked to study them. D. E. Smith says: "This ideal is not always easy to realize, but we are approaching it in our education of children, and the tendency is a healthy one."[2] The teacher is to cause the pupil to feel a need by taking advantage of the quantitative situations the pupil meets in his life outside of school and by so setting the stage that he will meet others. The latter plan is illustrated when the pupil undertakes a project in the manual-training shop or in the domestic-science laboratory and discovers that he needs arithmetic, or when arithmetic is taught incidentally. Need is also felt when a pupil experiences difficulty in controlling a situation efficiently. In such a case he needs a better method of control or drill upon his present method. The attempt to make arithmetical problems "real" and "concrete" has been prompted, in part, by the desire to secure motive.

Objective methods.—Prior to this period the use of objective materials had become rather indiscriminate, and often was looked upon as an end in itself. The significant feature of the objective teaching of this period has been a tendency toward a more refined correlation of the pupil's "experience with the social problem or subject involved." The objective materials have become more varied, and there is an increasing tendency to look upon them simply as a means to an end. These changes have resulted in a wider distribution of objective methods, but at the same time a clearer understanding of the function of objective materials has resulted in the total objective teaching being reduced.

[1] Strayer: A Brief Course in the Teaching Process, p. 180. [2] The Teaching of Arithmetic, p. 87.

Correlation.—When attention was focused upon the child the unitary nature of his life outside of school was revealed. In contrast, the course of study portioned out the child's time to the several school subjects, and each subject jealously guarded its apportioned period, resenting any encroachment. Each school subject was taught isolated from the other subjects. The topical arrangement of texts, as in the case of arithmetic, tended to isolate topics within a subject. And to a very considerable extent the work of successive days was isolated. Within the recent period plans for relieving this isolation have been proposed. Because of their bearing upon the teaching of arithmetic, some of them are worthy of our notice. The subcommittee of the Committee of Fifteen appointed by the National Education Association, 1893, reporting on the correlation of studies recognized five "staple branches of the elementary course of study." These were grammar, literature, arithmetic, geography, and history. They contended that "there should be rigid isolation of the elements of each branch."

In opposition to this, plans of concentration were proposed. A subject, or a closely related group of subjects, was taken as a center and all other school subjects were made subsidiary. A well-known attempt at concentration was made by Col. F. W. Parker at the normal school of Cook County, Ill. He concentrated the curriculum around the scientific subjects, elementary science, geography, myth, and history. Arithmetic was simply a means for controlling arithmetical situations within these subjects. By using it as a tool, arithmetic would be sufficiently learned. In fact Col. Parker believed that geography alone is sufficient.

If the child had no other study than that of geography, and the exercise of the numbering faculty met the necessities of the child's increasing knowledge, both of observation and imagination, the opportunities for the acquisition of the knowledge of arithmetic, as it is now understood, would be fully adequate.[1]

Charles A. McMurry, in *The Elements of General Method*, 1903, advocates correlation. This he defines as "such a connection between the parts of each study and such a spinning of relations and connecting links between different sciences that unity may spring out of the variety of knowledge." This is opposed to a plan of concentration such as proposed by Col. Parker. Each important study is to be isolated for purposes of instruction. But correlation also means that arithmetic is to be taught so that every important topic will be seen "in its natural relations to topics in other studies, thus binding the studies together in a multitude of close interrelations." In this way arithmetic, though taught as a separate subject, is to be correlated with geography, elementary science, history, etc. This is to be done by taking problems from

[1] Talks on Pedagogics, p. 71.

these subjects for part of the work in the arithmetic class and by using the knowledge learned in the arithmetic class as a tool for the better understanding on these other subjects.

By some, correlation was given additional meaning. Connection was to be made between topics within a subject, and even between the lessons of successive days. This was to be accomplished by a proper ordering of the topics and by reviews. To review the previous lesson to secure the connection became with many a necessary mark of good teaching.

Drill.—If one may draw conclusions from the texts, the emphasis upon drill as a factor of the teaching process has, in general, increased in this period. All of the more popular texts give much space to exercises for rapid drill. Some are in the form of special devices whose function is to assist the teacher in calling for combinations rapidly and in a variable order. A device which seems to be standard, but which appears in several variations, consists of a number surrounded by other numbers placed along some contour. By choosing appropriate numbers this device may be used for drill upon any of the fundamental operations. The device may be used directly from the text, or it may be transferred to the blackboard. In either case the teacher designates a number on the contour, and the pupils are to perform the required operation upon it. For example, if the process is division, the number in the center is 8, and the number designated on the contour is 72, the pupils are to give the quotient of 72 divided by 8.

Other plans for securing rapid drill upon the fundamental number facts are counting by twos, by threes, by fours, etc., adding numbers as the teacher writes them on the board or dictates them, and using drill cards which have exercises upon them. The pupils may be divided into groups for number contests, the group winning who does the work the most rapidly or the most accurately; or, instead of dividing the class into groups, all may work all the exercises, and scores be kept. At the end the total scores for the set of exercises are computed, the pupil making the highest score being the winner. It has been urged that some time each day be devoted to rapid drill. One authority states "about five minutes a day devoted to rapid oral work are sufficient to keep grammar-school pupils in practice." Besides this "rapid oral work," he contends that there should be a definite amount of rapid written work every day. The median per cent of time given to *strictly* drill work in arithmetic in 564 cities is as follows for the several grades:[1] First grade 43 per cent, second grade 50 per cent, third grade 53 per cent, fourth grade 47 per cent, fifth grade 39 per cent, sixth grade 31 per cent, seventh grade 22 per cent, and eighth grade 17 per cent. But notwithstand-

[1] W. A. Jessup, "Economy of Time in Arithmetic," Elementary School Teacher, Vol. XIV, p. 474.

ing this increased emphasis upon drill, skill is regarded less as a primary aim than heretofore. The function of drill is being better understood.

Scientific investigation and experimentation.—The pioneer in this field was J. M. Rice,[1] 1902, who attempted to evaluate the excellence of instruction in arithmetic by measuring the results of that teaching and to determine what factors contribute to superior results. He gave a test to 6,000 pupils in the fourth to eighth grades, inclusive. On the basis of the data obtained he eliminates as controlling factors home environment, size of classes, time of day which a class recites, age of pupils, time devoted to arithmetic, amount of home work required, method of teaching, and general qualifications of teachers, and concludes that the quality of the supervision is the controlling factor in determining the achievement of pupils in arithmetic.

The procedure of Rice's investigation is open to criticism as might be expected of a pioneer study, but it stimulated and inspired other scientific investigations and experimentation. The major problems attacked have been: (1) What is the nature of the product of instruction in arithmetic? (2) What factors are most effective in producing arithmetical abilities? (3) How to measure these abilities and to set standards of attainment in these abilities. (4) The determination of superior methods of instruction and courses of study by scientific experimentation. (5) A scientific analysis and study of the learning process as it occurs in the case of arithmetic. The most extensive work on these problems has been by S. A. Courtis,[2] who received his inspiration and stimulus from an investigation by C. W. Stone.[3] In addition to identifying elementary arithmetical abilities, which we have mentioned on p. 131, Courtis[4] has devised tests for measuring these abilities, and has set tentative standards of attainment in them. His standard practice tests and the manual which accompanies them represent the product of his study of methods of instruction and the learning process. The most significant feature is a plan for giving individual instruction to pupils when formed in classes.

At present there is much scientific investigation and experimentation which is resulting in an accumulating body of data which can be used as a basis for directing the development of arithmetic as a school subject in the decades to come. This is the most conspicuous tendency at present, and the indications are that future development will be made in this way.

[1] "Educational Research, A Test in Arithmetic," The Forum, XXXIV: 281-297. "Causes of Success and Failure in Arithmetic," The Forum, XXXIV: pp. 437-452.

[2] See bibliography for a list of his published material.

[3] Arithmetical Abilities and Some Factors Determining Them.

[4] Manual of Instructions for Giving the Courtis Standard Tests, Department of Cooperative Research, Detroit. Also the Courtis Standard Practice Tests, World Book Co.

Chapter XI.

SUMMARY.

The place of arithmetic in education.—During the ciphering book period arithmetic was a part of the school curriculum in those towns where it was demanded as a tool of commerce. In communities whose interests were not commercial and in the rural districts it was frequently not given a place in the plan of education and was conceded to possess little or no educational value. When arithmetic was taught under these conditions, it was simply as a concession to its practical utility. This early attitude was modified somewhat before the close of the ciphering book period. The commercial need for arithmetic had become more widespread and more universally recognized. When the Colonies became a free and independent Nation and a Federal currency was established, interest in arithmetic was greatly augmented. In 1729 the publication of the first arithmetic by an American author had passed unnoticed, but the appearance of Nicolas Pike's text in 1788 marked the beginning of interest in improving the subject matter of arithmetic which was manifested by the publication of many texts. By the beginning of the nineteenth century arithmetic had been given a place in the schools, though not one of first importance. There is some indication of the recognition of an educational value in addition to the practical value. But to Warren Colburn is due the credit for initiating in this country the movement which gave to arithmetic the place of first importance in the curriculum of the elementary school and which caused some to exalt it as a newly discovered "royal road" to learning.

Recently there has been a reaction from this extreme disciplinary conception of arithmetic and a return to arithmetic as a practical subject. But the meaning of practical is not that of the eighteenth century. Arithmetic now represents tools which the child needs to control his present and potential quantitative situations. These tools are to be organized in accord with the nature of the child and as the child works out methods for controlling these quantitative situations and organizes the arithmetical tools which he has acquired, arithmetic fulfills its disciplinary function in his education.

The content of arithmetic and its organization.—Two complementary tendencies are revealed in the modifications of the content of arithmetic. Practical demands and the desire of the arithmetician for a logically rounded-out science have caused subject matter to be added, and tradition has tended to keep subject matter which has once been added. New subject matter has been much more rapidly incorporated than obselete subject matter has been discarded. The most conspicuous change of emphasis has been in reference to the rule of three. Formerly it was *the* great topic, the "golden rule" of arithmetic. Now it has been reduced to the inconspicuous topic of proportion. Evolution has been reduced from an array of specific rules for roots up to the "squared square-cube root" until now only square root is frequently given. Some topics, such as permutations and combinations, position, and infinite series, have been transferred to more advanced courses in mathematics. Other topics, such as fellowship, certain tables of denominate numbers, much of exchange, tare and trett, alligation, duodecimals, annuities, etc., have been dropped as topics because the need which they satisfied no longer exists. On the other hand decimal fractions now occupy a much larger place. This has been due to the introduction of a decimal currency. In this way the relative importance of common fractions has been lessened, but they now occupy more space than formerly. More significant than the increased space given to fractions is the fact that they have been moved forward in the course.

The first great change in the subject matter of arithmetic came with the work of Warren Colburn. He introduced primary arithmetic and intellectual or mental arithmetic, gave a place of prominence to common fractions, and omitted the rule of three and other topics as such. Many of the omissions for which he took a stand have since been made, and others are at present being urged.

Arithmetic being a practical subject, the problems, for the most part, have been practical when they were introduced. As conditions changed, some problems were no longer practical. Tradition tended to keep these in the texts, the result being that our texts have contained a number of problems from obsolete or obsolescent situations. A few arithmetical puzzles have always found a place in our texts. When the disciplinary function of arithmetic was emphasized, the number of such puzzles was much increased, particularly in the mental arithmetics. Recently the force of tradition has been very much weakened, and there has been a tendency to reduce the number of arithmetical puzzles and to insist that practical problems be practical. These practical problems are to be drawn from a wide range of human activities and from the child's own life.

In the larger features of organization we have had many variations and combinations of the original topical plan and the more recent spiral plan. From a strictly topical organization we have come to a moderate spiral for grades one to four, followed by a transition to a topical organization in grades seven and eight. In the details of organization the logical, deductive order of the past has been replaced by an attempted psychological order. Here again credit is due Warren Colburn for making the break with the past by organizing his texts upon the inductive plan. Following Colburn there was a partial relapse to the old logical deductive order, but recently there has been a movement toward the form and spirit of Colburn's organization.

Methods of teaching arithmetic.—Before 1821 the teacher's function was to set "sums," tell rules, and pass upon the correctness of the pupil's work. This instruction was given to the pupils individually. After 1821 pupils were usually instructed in classes, and in practice the technique of dealing with pupils in classes became almost synonymous with methods of teaching. However, the concept of the function of the teacher was enlarged to include explaining the process and problems. Colburn and some others believed that the teacher should guide the pupil in developing his own rules. Some emphasis was placed upon drill, and much emphasis upon exact forms of analyses. Colburn's ideas concerning the teaching of arithmetic were as progressive as were his texts, but he failed to exert much direct influence upon the mode of teaching. Recently, starting with an analysis of the nature of the child, a clearer conception of the subject matter involved and the goal to be attained, more rational methods of teaching arithmetic are being worked out. In these rational methods direct instruction and drill have a place. Motive by appeal to artificial incentives has been supplemented by motive secured by interest and by need. The spirit of present-day methods is to assist the child by making the instruction coincide with the natural working and development of the child's mind. Although Warren Colburn has influenced the present methods of teaching arithmetic scarcely at all, yet we are distinctly returning to the spirit and form of his methods. We are now, like him, studying the child for the basis of our methods.

The men who have made our arithmetic.—Warren Colburn without a doubt occupies first place, because of his influence in stimulating and directing the development. Much of our present arithmetic we owe to him directly or indirectly. He himself was much greater than his influence has been, and his writings are still sources of information as well as inspiration. To Joseph Ray we should give second place. He was not a great constructive writer and thinker as was Colburn, but his greatness consists rather of his ability to write clearly, to organize, and to adapt. Because he could do these things well, his texts have been given a wide and long-continued use in our schools. Following these two men, there are many others who have materially contributed to the molding of our present arithmetic and the methods of teaching it.

Some inferences.—The story of human activity, human progress, is always interesting, and it may be of value to the present generation in their attempts for improvement. The story of the development of arithmetic which we have traced repeatedly suggests that permanent improvement of content, organization, or methods of teaching must be based upon a clear conception of the child. In this was Colburn's greatness, and here also is the foundation of our recent progress.

In their enthusiasm to improve arithmetic and its teaching, the teachers have not maintained a critical attitude toward proposed reforms. Using methods which have since been shown to be fundamentally wrong, they have secured results which they interpreted as an improvement over previous results. The judgment of results has often been based upon biased opinions and has seldom been the result of a clear comprehension of the aim of arithmetic teaching and a comprehensive survey of the effect of the teaching upon the pupils. For this reason the judgments have at times been defective. But the fact remains that the belief of a teacher in a method has been a large factor in the determination of the measure of its success.

APPENDIX.

TABLE OF CONTENTS OF DILWORTH'S SCHOOLMASTER'S ASSISTANT.

PART I.—OF WHOLE NUMBERS.

PART II.—OF VULGAR FRACTIONS.

PART III.—OF DECIMAL FRACTIONS.

PART IV.—QUESTIONS.

PART V.—OF DUODECIMALS.

TABLE OF CONTENTS OF A NEW AND COMPLETE SYSTEM OF ARITHMETIC BY NICOLAS PIKE.

TABLE OF CONTENTS OF WARREN COLBURN'S FIRST LESSONS.[1]

PART I.

PART II.

TABLE OF CONTENTS OF WARREN COLBURN'S SEQUEL.

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[1] Colburn does not give a table of contents in the First Lessons, and this has been made from a study of the material given in the various sections.

XVII. To divide a fraction by a whole number. To multiply a fraction by a fraction.

XVIII. To multiply a fraction by dividing the denominator. Two ways to multiply and two ways to divide a fraction.

XIX. Addition and subtraction of fractions. To reduce them to a common denominator. To reduce them to lower terms.

XX. Contractions in division.

XXI. How to find the divisors of numbers. To find the greatest common divisor of two or more numbers. To reduce fractions to their lowest terms.

XXII. To find the least common multiple of two or more numbers. To reduce fractions to the least common denominator.

XXIII. To divide a whole number by a fraction, or a fraction by a fraction, when the purpose is to find how many times the divisor is contained in the dividend. To find the ratio of a fraction and a whole number, or of two fractions.

XXIV. To divide a whole number by a fraction, or a fraction by a fraction; a part of a number being given to find the whole. This is on the same principle as that of dividing a number into parts.

XXV. Decimal fractions. Numeration and notation of them.

XXVI. Addition and subtraction of decimals. To change a common fraction to a decimal.

XXVII. Multiplication of decimals.

XXVIII. Division of decimals.

XXIX. Circulating decimals. Proof of multiplication and division by casting out 9's.

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Judd, Sylvester. History of Hadley.

Keyser, Naaman H., *and others.* History of Old Germantown.

Orcutt, Samuel, *and* Beardsley, Ambrose. History of the Old Town of Derby, Connecticut.

Orcutt, William Dana. Good Old Dorchester.

Schenck, E. H. The history of Fairfield, Connecticut.

Slafter, Carlos. The schools and teachers of Dedham, Massachusetts.

Watson, John F. Annals of Philadelphia.

Winsor, Justin, *ed.* Memorial history of Boston.

TEXTBOOKS OF THE PERIOD.[1]

Adams, Daniel. The scholar's arithmetic or federal accountant, 1801.[2] 10th ed. 1815.

Has blank spaces for the solution of the problems and examples by the pupil.

[1] Only a few of the most significant texts are listed. For a complete list the reader is referred to the bibliography by Greenwood and Martin.

[2] The first date following the title of a book is the date of publication of date of first copyright. Other dates refer to revision.

American tutor's assistant or a compendious system of practical arithmetic. 3d ed. 1797.

"Arithmetic is the art of computing by numbers."

Cocker, Edward. Cocker's vulgar arithmetic 1677.

An American edition of this English text was published in Philadelphia in 1779. It has been said that this text "may be considered the father of modern arithmetic, as it furnished the plan all of them have copied."

Daboll, Nathan. Daboll's schoolmaster's assistant. 1799.

This became a very popular text.

Dilworth, Thomas. The schoolmaster's assistant: Being a compendium of arithmetic, both practical and theoretical. 1743.

Numerous editions of this English text were printed in this country. A revision by William Hawley was published in 1807 which passed through five editions by 1817. Robert Patterson edited an edition in 1805 and William Stoddard another in 1817.

Federal ready reckoner: or trader's valuable guide in purchasing and selling all kinds of articles by wholesale and retail. Printed by Leonard Worcester, 1795.

A book of tables for computating the cost of articles and interest.

Fisher, George. The instructor, or, American young man's best companion. 30th ed. 1785.

See page 40 for complete title which indicates the nature of the book.

Greenwood, Isaac. Arithmetic vulgar and decimal, with application thereof to a variety of cases in trade and commerce. 1729.

This is the first arithmetic by an American author. There are copies in the Harvard College Library, the Boston Public Library, and the Library of Congress.

Hodder, James. Hodder's arithmetic: or that necessary art made most easy 1661.

The 28th edition of this English text was printed in Boston in 1719. This was the "first purely arithmetical work published in the United States."

Kendal, David. The young lady's arithmetic. 1797.

"Arithmetic is the art of computing by figures." Directs the student to "learn well by heart" rules and definitions. It is simply a condensed form of the arithmetics of this period.

Kimber, Emmor. Arithmetic made easy to children: Being a collection of useful and familiar examples, methodically arranged and under their respective heads. 2d ed. 1805.

A traditional text.

Leavitt, Dudley. Elements of arithmetic made easy by an original introduction to that science. 1813.

Lee, Chauncey. The American accomptant. 1797

This is the first arithmetic in which a dollar mark is used and it is also interesting because of an attempt by the author to decimalize all weights and measures.

M'Donald, Alexander. The youth's assistant: being a plain easy, and comprehensive guide to practical arithmetic. 1785, 1789.

The practical reasons for teaching arithmetic are emphasized.

Pike, Nicolas. A new and complete system of arithmetic, composed for the use of the citizens of the United States. 1788.

This is commonly supposed to be the second arithmetic by an American author but probably was the fifth. It is the first arithmetic in which the Federal money is given.

Pike, Stephen. The teacher's assistant. 1811.

It was revised in 1845 and 1852.

Root, Erastus. Introduction to arithmetic, for the use of common schools. 1795.

Sarjeant, Thomas. Elementary principles of arithmetic; with their application to the trade and commerce of the United States. 1788.

Temple, Samuel. An arithmetical primer for young masters and misses; containing simply, the first principles of that most useful art. 1809. 48 pp.

It is intended to be followed by a more advanced book. Blank spaces are left for the solution of examples.

Vinall, John. Preceptor's assistant. 1792.
Vyse, Charles. A key, or the arithmetician's repository. 9th ed. 1806.
A key to the tutor's guide.

Walkingame, Francis. The tutor's assistant; being a compendium of arithmetic and a complete question book.
A text by an English author reprinted in this country.

Walsh, Michael. A new system of mercantile arithmetic; adapted to the commerce of the United States, in its domestic and foreign relations; with forms of accounts, and other writings usually occurring in trade. 1801.

Watt, Alexander. A new plain and systematic compendium of arithmetic adapted to the commerce of the United States. 1806, 1814.

Welch, Oliver. American arithmetic. 1814.

White, John. A practical system of mental arithmetic, or a new method of making calculations by the action of the mind without pen, ink, pencil, or paper. 1818.
Rules and facts were to be memorized. Many short-cuts are given but there are no analyses and the book is not a "mental" arithmetic in the sense the term is generally used.

Wingate, Edmund. Artificial arithmetic. 1629.
The 15th edition of this English text was published in 1720. Littlefield states: "A copy of the first edition is now in existence which was used in the Winslow family of Massachusetts for over one hundred years."

Workman, Benjamin. A treatise of arithmetic in theory and practice; containing everything important in the study of abstract and applicate numbers. 1788.
This is an edition of an English text by John Gough. It was revised by R. Patterson in 1796.

The youth's instructor in the English tongue. 1760.

OTHER SOURCES.

Barnard, Henry. American journal of education.
For articles bearing upon this period see Analytical Index to Barnard's American Journal of Education under "Reminiscences of schools and teachers."

Burton, Warren. The district school as it was.
A very interesting description of schools in the period from 1810 to 1830.

Cajori, F. The teaching and history of mathematics in the United States. Washington, Government Printing Office. 1890. (U. S. Bureau of Education. Circular of information, No. 3, 1890.)
A general account devoted mostly to mathematics beyond arithmetic.

Clews, Elsie W. Educational legislation and administration of Colonial governments.

Colesworthy, D. C. John Tileston's school.

Cornell, William Mason. The history of Pennsylvania.

Cyphering books.
Only a few of these manuscripts are available in libraries. Two were found in the library of the American Antiquarian Society and two in the library of the Worcester Society of Antiquity. In addition, a manuscript by Miss Catherine G. Willard, now in possession of Prof. W. C. Gore, of the University of Chicago, was examined.

Evans, Charles. Evans' American bibliography.
A complete bibliography of all books and pamphlets published in America beginning with the first book printed in America in 1639.

Jackson, L. L. The educational significance of sixteenth century arithmetic. 1906.

Johnson, Clifton. Old-time schools and school-books.
A very interesting account.

Kilpatrick, William H. The Dutch schools of New Netherland and Colonial New York. Washington, Government Printing Office, 1912. (U. S. Bureau of Education. Bulletin, No. 12, 1912.)
A carefully prepared account of an intensive study.

Littlefield, G. E. Early New England schools, club of old volumes. Boston, 1904.

Meriwether, Colyer. Our Colonial curriculum, 1607–1776.

Murry, David. History of education in New Jersey. Washington, Government Printing office, 1899. (U. S. Bureau of Education. Circular of Information, No. 1, 1899.)

Powell, Lyman C. The history of education in Delaware. Washington, Government Printing Office, 1893. (U. S. Bureau of Education. Circular of Information, No. 3, 1893.)

Steiner, B. C. History of education in Maryland. Washington, Government Printing Office, 1894. (U. S. Bureau of Education. Circular of Information, No. 2, 1894.)

Tolman, William H. History of higher education in Rhode Island. Washington, Government Printing Office, 1894. (U. S. Bureau of Education. Circular of Information, No. 1, 1894.)

Watson, F. The beginnings of the teaching of modern subjects in England.

——— The English grammar schools to 1660, their curriculum and practice.

This and the above book by the same author give a good account of contemporary practice in England.

Wickersham, J. P. History of education in Pennsylvania.

Worcester Society of Antiquity. Collections, vol. 9, Proceedings, 1888-1890.

THE BEGINNING OF THE PESTALOZZIAN MOVEMENT IN AMERICA AS APPLIED TO ARITHMETIC.

Barnard's American Journal of Education, vol. 2, p. 294.

This appears to be the most complete biographical account of Colburn available.

Biber, E. Henry Pestalozzi and his plan of education.

Colburn, Warren. First lessons in arithmetic on the plan of Pestalozzi; with some improvements. 1821.

An edition of 1826 has the title, "Colburn's first lessons. Intellectual arithmetic upon the inductive method of instruction." This title has been retained. A copy of the first edition is in the library of the American Antiquarian Society, Worcester.

——— Arithmetic; being a sequel to first lessons in arithmetic. 1822.

——— The teaching of arithmetic. An address delivered before the American Institute of Instruction in Boston, August. 1830.

It was published in the proceedings of that society and was reprinted in the Elementary School Teacher, June, 1912.

Krüsi, Herman. Pestalozzi: His life, work, and influence.

One section of the book is devoted to the "Principles and method of Pestalozzi." A chapter on "Special application of Pestalozzi's method" gives Pestalozzi's ideas on number.

Monroe, Will S. History of the Pestalozzian movement in the United States.

This account deals only with general features. No mention is made of Warren Colburn and very little is said concerning the influence of Pestalozzian principles upon school subjects and the method of teaching.

Pestalozzi, J. H. Anschauungslehre der Zahlenverhaltnisse, Part I.

This contains a list of exercises upon Pestalozzi's unit's table. Parts II and III contain those on the fraction tables. Herman Krüsi assisted in preparing the exercises.

——— Leonard and Gertrude. Trans. by Eva Channing. Boston, D. C. Heath & Co.

A narrative account of Pestalozzi's theory of education before 1781.

——— How Gertrude teaches her children. Trans. by Lucy E. Holland and Frances C. Turner. Syracuse, N. Y., C. W. Bardeen.

An excellent account of Pestalozzi's educational theory.

Ünger, Fredrich. Die Methodik der Praktischen Arithmetik.

A reliable account of the development of arithmetic as a school subject in Germany.

FROM 1821 TO 1892.

GENERAL SOURCES.

American Journal of Education, vol. 1, 1826.

Connecticut school document, Nos. 7 and 16.

These contain reports of surveys of Connecticut schools.

Davies, Charles. The logic and utility of mathematics. 1850.
"A full analysis of the science of mathematics."

DeGraff. The schoolroom guide. 1877.

Fowle, William B. The teacher's institute; or familiar hints to young teachers. 1866.
A book on the teaching of the common branches.

Greenwood, J. M. Principles of education practically applied. 1887.

Key to Ray's practical arithmetic. 1845.
The teaching of arithmetic is discussed briefly.

Page, David P. Theory and practice of teaching. 1847.

Stoddard, John F. Methods of teaching arithmetic and key to Stoddard's American mental arithmetic.

ARITHMETIC TEXTS

Some of the more important series of arithmetics have been described in Chapter VIII. The authors of these series are included in this list and a page reference is given to Chapter VIII. No attempt has been made to give a complete list of the arithmetic texts published during the period. Those interested in such a list are referred to the bibliography by J. M. Greenwood and Artemas Martin.

Adams, Daniel. Adams's new arithmetic. 1827. See p. 103.

Adams, Frederick A. Common and high school arithmetic. 1846.

Belfield, H. H. The revised model elementary arithmetic. 1887.
"Oral exercises will be found on the left-hand page and written on the right, with a few exceptions."

Brooks, Edward. See p. 102.

Chase, Pliney E., *and* Mann, Horace. See p. 103.

Cobb, Lyman. Arithmetic. 1832.

Colburn, Dana P. The child's book of arithmetic, 1859; Intellectual arithmetic, 1859; Common school arithmetic, 1858; Arithmetic and its applications, 1855.

Crozet, Claudius. An arithmetic for colleges and schools. 1848.

Davies, Charles. See p. 95.

Dean, Philotus. Primary arithmetic, 1860; Intellectual arithmetic, 1861; Elementary arithmetic, 1868; Intermediate arithmetic, 1865.

Eaton, James Stewart. See p. 99.

Emerson, Fredrick. See p. 94.

Ernst, Henry. Arithmetic. 1857.

Felter, S. A. Primary arithmetic, 1863; New primary, 1880; First lessons, 1865, 1880; Intermediate arithmetic, 1862, 1866; New intermediate, 1875; New practical, 1872; Complete arithmetic, 1877; Advanced arithmetic, 1877

Fowle, William B. The child's arithmetic, or the elements of calculation, in the spirit of Pestalozzi's method. 1826.
For the use of children between the years of three and six.

French, John H. First lessons in numbers, 1866; Mental arithmetic, 1870; Elementary arithmetic, 1867; Common school arithmetic, 1869.

Goff, M. B. - First book in arithmetic, 1876; Arithmetical problems, 1877; Elementary arithmetic, 1888.

Goodrich, S. G. Peter Parley's method of teaching arithmetic to children. 1833. See p. 103.

Greenleaf, Benjamin. See p. 97.

Groesbeck, John. Commercial arithmetic. 1868.

Hagar, Daniel B. Primary lessons in numbers, 1871; Elementary arithmetic, 1871; Dictation problems and reviews in arithmetic, 1874.

Hanna, J. S. Complete ready reckoner. 1871.

Harper and Bros. Two-book series. 1882.

Hill, Thomas. An elementary arithmetic. 1845.

"It is not the design of this treatise to make arithmetic easy nor to give such practical skill as may be useful in business."

Hoose, James H. Arithmetic. 1882.

The author attempted to follow Pestalozzi's plan.

Jameson, Richard S. The basis of arithmetic. 1867.

Kirk, Alfred. Model elementary arithmetic, 1876; Model complete arithmetic, 1875.

Leach, Daniel, *and* Swan, Robert. Elementary intellectual arithmetic. 1853.

Leonard, George, *Jr.* A practical treatise on arithmetic. 1839.

Loomis, Elias. The elements of arithmetic. 1863.

Mecutchen, Samuel. Graded problems in arithmetic and mensuration. 1880.

—— *and* Sayre, George M. New American practical arithmetic. 1877.

Milne, W. J. First lessons, 1878; Practical arithmetic, 1878; Elementary arithmetic, 1882.

Olney, Edward. Primary arithmetic, 1874; Elements of arithmetic, 1875; Practical arithmetic, 1879; First lessons in arithmetic, 1881; Teacher's handbook of arithmetical exercises, 1875.

Olney, Jesse. An improved system of arithmetic. 1839.

Orton, Hoy D. Lightning calculator. 1866.

Parke, Uriah. Farmers', merchants', and mechanics' practical arithmetic. 3d ed., 1839; 20th ed., 1847.

Payson, John P. Practical arithmetic. 1887.

Peck, William G. Complete arithmetic. 1874.

Perkins, George Robert, *and* Quackenbos, George P. Builded on the texts of Perkins. See p. 103.

Raub, Albert N. Elementary arithmetic, 1877; Complete arithmetic, 1877.

Ray, Joseph. See p. 96.

Robinson, James, *Jr.* The American arithmetic, 1825; Elementary lessons in intellectual arithmetic, 1830.

Robinson, H. N., *also* Fish, D. W., who was associated with Robinson. See p. 101.

Rodgers, M. H. A notebook of arithmetical rules and definitions. 1862.

Ruger, William. A new system of arithmetic. 1829.

Sadler, W. H., *and* Will, W. R. Inductive arithmetic. 1885.

Sanford, Shelton Palmer. The intermediate arithmetic, 1871. The common-school arithmetic, 1872.

Seymour, George E. Elementary arithmetic, 1880; Practical arithmetic, 1880.

Singer, Edgar A. Elementary arithmetic, 1876; The model mental arithmetic, 1876.

Smiley, Thomas T. The new federal calculator. 1828.

"In appearance is a twin to Daboll's arithmetic." It was still printed in 1899 by J. B. Lippincott & Co. It seems to have been revised by L. Talbot and printed in 1841 under the title, "The Western practical arithmetic."

Smith, Roswell C. The little federal calculator. 1832. (It starts, "How many little fingers have you on your right hand? How many on both?"); Practical and mental arithmetic, designed principally to accompany Daboll's system of arithmetic, 1826; Practical and mental arithmetic on a new plan, 1827; The productive arithmetic, 1841.

Stewart, Charles. An analytical arithmetic for the use of schools and colleges. 1856, 1863, 1874.

Stoddard, John F. See p. 99.

Talbot, John L. The Western practical arithmetic. 1841.

Thomson, James B. See p. 98.

Ticknor, Almon. Columbian calculator. 1846.

Towne, Paul Allen. Mental arithmetic. 1870.

Townsend, Julius L. Primary arithmetic, 1885; Intermediate arithmetic, 1885; Grammar-school department, 1885.

Tracy, Calvin. Commercial and mechanical arithmetic. 1851.
Vogdes, William. The United States arithmetic. 1845.
Walton, George A. See p. 103.
Wentworth, G. A. The first steps in number, 1884; Grammar-school arithmetic, 1885; High-school arithmetic, 1881; Primary arithmetic, 1891.
—— and Reed, E. M. The first steps in number. 1884.
This text and the Primary arithmetic by G. A. Wentworth are upon the Grube method.

Wheeler, H. N. Second lessons in arithmetic. 1888.
"I have tried in all cases to follow the inductive method of instruction as illustrated in Warren Colburn's First Lessons, a book which has confessedly done more for the cause of education than any other textbook that has ever been published." (From the preface.)

White, E. E. A primary arithmetic, 1868; An intermediate arithmetic, 1870.

RECENT TENDENCIES AND DEVELOPMENTS.

There is no attempt to give a complete bibliography of recent scientific studies, since this has been done by Mr. Howell, whose work is mentioned below.

Bagley, W. C. Educational values. Chapter 12.
This gives a brief historical account of the reaction against formal discipline.

Brown, C. E. Psychology of the simple arithmetical processes. American Journal of Psychology, 17:1, 1906.
Brown, J. C., and Coffman, Lotus D. How to teach arithmetic. 1914.
Courtis, S. A. Teachers' manual and standard practice tests. 1914.
—— Manual of instructions for giving and scoring the Courtis standard tests. Rev. ed. 1914.
—— Measurement of growth and efficiency in arithmetic. Elementary School Teacher, 10:55-74, 177-99; 11:171-85, 360-70, 528-39; 12:172-37.
—— Standard scores in arithmetic. Elementary School Teacher, November, 1911.
—— Reliability of single measurements with standard tests. Elementary School Teacher, March, 1913.
Eliot, Charles W. Educational reform.
This is a series of addresses extending over a number of years. They reflect progressive tendencies.

Hart, W. W. Community arithmetic for seventh and eighth grades. Elementary School Teacher, 11:285-95.
Howell, Henry Budd. A foundation study in the pedagogy of arithmetic. New York, Macmillan Company, 1914.
"A very complete résumé of the extensive experimental literature" on arithmetic. The book contains an extensive bibliography.

International Commission on the Teaching of Mathematics. Report of the American Commissioners. Washington, Government Printing Office, 1912. (U. S. Bureau of Education Bulletin, No. 14, 1912.)
A summary for the elementary school is given on pages 14-25.

Jessup, W. A. Grade for introduction of a text in arithmetic. Elementary School Journal, 15:163, November, 1914.
—— Economy of time in arithmetic. Elementary School Teacher, 14:461-76, June, 1914.

McMurry, Charles A. Special method in arithmetic. 1905.
Herbartian doctrines applied to arithmetic.

—— The elements of general method, 1903.
A chapter is given to correlation.

McLellan, James A., *and* Dewey John. The psychology of number. 1895.

A constructive interpretation of the nature of the child with respect to number and its applications to the teaching of arithmetic.

Mathematics in the elementary schools of the United States. The American report of Committees I and II of the International Commission on the Teaching of Mathematics. Washington, Government Printing Office. 1911. (U. S. Bureau of Education.)

A comprehensive survey.

Matthews, Pauline E. Report of experiment on the value of plays and games in arithmetical drill. Elementary School Teacher, 14:319-324.

Monroe, Walter S. A report of the use of the Courtis standard research tests in arithmetic in 24 cities. Topeka, Kans., 1915. 94 pp. (Kansas State normal school, Emporia. Bulletin. New series, vol. 4, no. 8.)

National Education Association. Committee of Fifteen. Report on the correlation of studies. 1895.

—— Committee of Ten. Report.

—— Proceedings.

These proceedings reflect the educational thought of the teachers of the United States. The following have particular reference to arithmetic: 1880, p. 114; 1881, p. 21, 85, 106. 1892, p. 617, 620. 1895, p. 264, 300.

Newcomb, Simon. Methods of teaching arithmetic. Educational Review, 31:339-350.

Parker, Francis W. Talks on pedagogics.

Colonel Parker gives a plan for making the scientific subjects a center around which arithmetic and other form subjects may be correlated.

Patridge, Lelia E. The "Quincy Methods." Illus. 1885.

Rice, J. M. Educational research: a test in arithmetic. Forum, 34: 281-297.

—— Causes of success and failure in arithmetic. Forum, 34: 437-452.

Smith, D. E. Handbook to Smith's arithmetics. 1905.

—— The teaching of arithmetic. 1909.

—— The teaching of elementary mathematics. 1900

Stamper, A. W. A textbook on the teaching of arithmetic. 1913.

Starch, D. Transfer of training in arithmetical operations. Journal of Educational Psychology, 1911.

Stone, C. W. Arithmetical abilities and some factors determining them. 1908.

A report of an experimental investigation.

Suzzallo, Henry. The teaching of primary arithmetic. 1912.

A critical study of recent tendencies in method

Training of teachers of elementary and secondary mathematics. Washington, Government Printing Office, 1911. (U. S. Bureau of Education. Bulletin, No 12. 1911.)

Walsh, John H. Practical methods in arithmetic 1911.

Wilson, G. M. Course of study in mathematics. Connersville, Ind., public schools 1911.

A course of study worked out by Supt. Wilson and his teachers after examining many courses of study and texts.

Winch, W. H. Accuracy in school children. Does improvement in numerical accuracy transfer to arithmetical reasoning? Journal of Educational Psychology, 1910.

—— Further work on numerical accuracy in school children. Does improvement in numerical accuracy transfer? Journal of Educational Psychology, 1911.

Young, J. W. A. The teaching of mathematics. 1906.

REPRESENTATIVE TEXTS OF THE PERIOD.

Belfield, H. H., *and* Brooks, Sarah C. The rational arithmetics. 1898.

Hall, Frank H. The Werner arithmetics; a three-book course for graded schools. 1896.

——— The Hall arithmetics, a two-book course for graded or ungraded schools. 1896.

These arithmetics represent the extreme spiral arrangement.

Hamilton, Samuel. Hamilton's arithmetic. 1907.

McLellan, J. A., *and* Ames, A. F. The primary public school arithmetic. 1898.

——— The public school arithmetic. 1902.

These texts are based upon McLellan and Dewey's psychology of number.

Myers, George W. Myers's arithmetics. 1903.

Seeley, Levi. Elements of the Grube method. 1890.

Smith, D. E. Smith's arithmetics. 1904.

Southworth, G. A., *and* Stone, J. C. The Southworth-Stone arithmetics. 1904.

Speer, W. W. Form lessons. 1888.

——— A series of arithmetics.

The first book of the series was published in 1896.

Stone, J. C., *and* Millis, J. F. The Stone-Millis arithmetics. 1910.

Walsh, John H. Walsh's arithmetics. 1895.

Wentworth, G., *and* Smith, D. E. The Wentworth-Smith arithmetics. 1909.

Young, J. W. A., *and* Jackson, L. L. The Appleton arithmetics. 1909.

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